Math 1553 Worksheet §2.2, 2.3, 3.1, 3.2

1. Find the parametric form for the general solution to the following system of equations, if such a solution exists:

$$\begin{aligned} x_1 + \ x_2 + x_3 - \ x_4 &= -3 \\ 2x_1 + 3x_2 + x_3 - 5x_4 &= -9 \\ x_1 + 3x_2 - x_3 - 6x_4 &= \ 7 \end{aligned} \quad \rightsquigarrow \quad (x_1, x_2, x_3, x_4) = (?, ?, ?, ?).$$

2. Write $\begin{pmatrix} 6 \\ 11 \\ 6 \end{pmatrix}$ as a linear combination of the vectors

$$u = \begin{pmatrix} 2 \\ 1 \\ 4 \end{pmatrix} \qquad v = \begin{pmatrix} 1 \\ -1 \\ 3 \end{pmatrix} \qquad w = \begin{pmatrix} 3 \\ 2 \\ 5 \end{pmatrix}.$$

3. Decide if each of the following statements is true or false. If it is true, prove it; if it is false, provide a counterexample.

a) Every set of four or more vectors in $\mathbf{R}^3$ will span $\mathbf{R}^3$.

b) The span of any set (including the empty set!) contains the zero vector.