

## Math 1553 Worksheet §2.2

1. True or false (justify your answer): If a system of linear equations has more variables than equations, it must be consistent.
2. a) Which of the following matrices are in row echelon form? Which are in reduced row echelon form?  
b) Which entries are the pivots? Which are the pivot columns?

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{pmatrix} \quad \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

3. a) Row reduce the following matrices to reduced row echelon form.  
b) If these are augmented matrices for a linear system (with the last column being after the = sign), then which are inconsistent? Which have a *unique* solution?

$$\begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 7 \\ 6 & 7 & 8 & 9 \end{pmatrix} \quad \begin{pmatrix} 1 & 3 & 5 & 7 \\ 3 & 5 & 7 & 9 \\ 5 & 7 & 9 & 1 \end{pmatrix} \quad \begin{pmatrix} 3 & -4 & 2 & 0 \\ -8 & 12 & -4 & 0 \\ -6 & 8 & -1 & 0 \end{pmatrix}$$

4. What would you have to know about the pivot columns in an augmented matrix in order to know that the corresponding linear system is consistent and has a unique solution?
5. Is there a degree-three polynomial  $P(x)$  whose graph passes through the points  $(-2, 6)$ ,  $(-1, 4)$ ,  $(1, 6)$ , and  $(2, 22)$ ? If so, how many degree-three polynomials have a graph that through those four points? We will answer this question in steps below.
  - a) If  $P(x) = a_0 + a_1x + a_2x^2 + a_3x^3$  is a degree-three polynomial passing through the four points listed above, then  $P(-2) = 6$ ,  $P(-1) = 4$ ,  $P(1) = 6$ , and  $P(2) = 22$ . Write a system of four equations which we would solve to find  $a_0$ ,  $a_1$ ,  $a_2$ , and  $a_3$ .
  - b) Write the augmented matrix to represent this system and put it into reduced row-echelon form. Is the system consistent? How many solutions does it have?