Math 1553 Worksheet §3.5

Linear Independence: Concept Questions

1. If three vectors v_1, v_2, v_3 span $\mathbf{R}^3$, must those vectors be linearly independent? Why or why not?

2. Which of the following true statements can be checked without row reduction?

a) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ \pi \end{pmatrix}, \begin{pmatrix} 0 \\ \sqrt{2} \\ 0 \end{pmatrix} \right\}$ is linearly independent.

b) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix} \right\}$ is linearly independent.

c) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 5 \\ 7 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \right\}$ is linearly dependent.

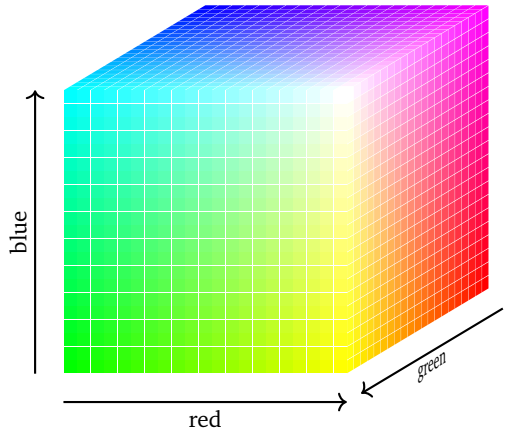
d) $\left\{ \begin{pmatrix} 3 \\ 3 \\ 4 \end{pmatrix}, \begin{pmatrix} 0 \\ 10 \\ 20 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} \right\}$ is linearly dependent.

3. How many solutions can the matrix equation $Ax = b$ have if the columns of A are linearly independent? [Try $b = 0$ first.]

a) 0 **b)** 1 **c)** ∞.

Linear Independence: Additive Color Theory

Every color on my computer monitor is a vector in $\mathbf{R}^3$ with coordinates between 0 and 255, inclusive. The coordinates correspond to the amount of red, green, and blue in the color.



Given colors $v_1, v_2, \ldots, v_p$, we can form a "weighted average" of these colors by making a linear combination

$$v = c_1 v_1 + c_2 v_2 + \cdots + c_p v_p$$

with $c_1 + c_2 + \cdots + c_p = 1$. Example:

$\frac{1}{2}$  $+$ $\frac{1}{2}$  $=$ 

4. Consider the colors on the right. Are these colors linearly independent? What does this tell you about the colors?

$\begin{pmatrix}240\\140\\0\end{pmatrix}$ $\begin{pmatrix}0\\120\\100\end{pmatrix}$ $\begin{pmatrix}60\\125\\75\end{pmatrix}$

5. Consider the colors on the right. For which h is

$$\left\{\begin{pmatrix}180\\50\\200\end{pmatrix}, \begin{pmatrix}100\\150\\100\end{pmatrix}, \begin{pmatrix}116\\130\\h\end{pmatrix}\right\}$$

linearly dependent? What does that say about the corresponding color?

$\begin{pmatrix}180\\50\\200\end{pmatrix}$ $\begin{pmatrix}100\\150\\100\end{pmatrix}$

$h =$ 40 80 120 160 200 240