Math 1553 Worksheet §3.6, 3.7, 3.8

1. Find bases for the column space and the null space of

$$A = \begin{pmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{pmatrix}$$

2. Consider the following vectors in $\mathbf{R}^3$:

$$b_1 = \begin{pmatrix} 2 \\ 2 \\ 2 \end{pmatrix} \qquad b_2 = \begin{pmatrix} 1 \\ 4 \\ 3 \end{pmatrix} \qquad u = \begin{pmatrix} 1 \\ 10 \\ 7 \end{pmatrix}$$

Let $V = \text{Span}\{b_1, b_2\}$.

a) Explain why $\mathcal{B} = \{b_1, b_2\}$ is a basis for V.

b) Determine if u is in V. If it is, find $[u]_{\mathcal{B}}$, the $\mathcal{B}$-coordinate vector of u.

c) Find a vector b_3 such that $\{b_1, b_2, b_3\}$ is a basis of $\mathbf{R}^3$.

3. Answer "yes" if the statement is always true, "no" if it is always false, and "maybe" otherwise.

a) If A is a 3×100 matrix of rank 2, then $\dim \operatorname{Nul} A = 98$.

b) If A is an $n \times n$ matrix and $\operatorname{Col} A = \mathbf{R}^n$, then $Ax = 0$ has a nontrivial solution.

c) If A is an $m \times n$ matrix and $Ax = 0$ has a nontrivial solution, then the columns of A form a basis for $\mathbf{R}^m$.

d) The empty set is a subspace of $\mathbf{R}^m$.

4. Which of the following are subspaces of $\mathbf{R}^4$? Why or why not?

a) $V = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; x + y = 0 \text{ and } z + w = 0 \right\}$

b) $W = \left\{ \begin{pmatrix} x \\ y \\ z \\ w \end{pmatrix} \text{ in } \mathbf{R}^4 \;\middle|\; xy - zw = 0 \right\}$