Math 1553 Worksheet §6.5

1. Let A and B be 3×3 real matrices. Answer yes / no / maybe:

a) Can the eigenvalues of A be $3, 5$, and $2 + i$?

b) Can A have three complex (non-real) eigenvalues?

c) Can A have a complex eigenvalue with multiplicity 2?

d) Suppose that A has one eigenvalue of algebraic multiplicity 3. Is A diagonalizable?

e) Suppose that A has two distinct eigenvalues. Is it diagonalizable?

f) Suppose that A has three distinct eigenvalues. Is it invertible?

g) If A and B both have eigenvalues $-1, 0, 1$, then A is similar to B.

h) If A and B have the same eigenvalues, then A is similar to B.

i) If A and B have one real and one complex eigenvalue in common, then A is similar to B.

2. Let $A = \begin{pmatrix} 1 & 2 \\ -2 & 1 \end{pmatrix}$.

a) Find all (complex) eigenvalues and eigenvectors of A.

b) Write $A = PCP^{-1}$, where C is a rotation followed by a scale.

c) What does A do geometrically? Draw a picture.

3. Let $A = \begin{pmatrix} 4 & -3 & 3 \\ 3 & 4 & -2 \\ 0 & 0 & 2 \end{pmatrix}$.

a) Find all (complex) eigenvalues and eigenvectors of A.

b) Write $A = PCP^{-1}$, where C is a block diagonal matrix, as in the slides for 11/9.

c) What does A do geometrically? Draw a picture.