

Math 1553 Worksheet §§7.2–7.5

1. a) Find the standard matrix A for proj_W , where $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}, \begin{pmatrix} 3 \\ -1 \\ 2 \end{pmatrix} \right\}$.

b) Find the standard matrix B for proj_L , where $L = \text{Span} \left\{ \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix} \right\}$.

c) Answer the following questions without doing any calculations:

(1) What are A^2 and B^2 ?

(2) What are A^{-1} and B^{-1} ?

(3) What are AB and BA ?

(4) Is A or B diagonalizable?

(5) What are the eigenvalues of A and B ? What are their algebraic multiplicities?

(6) Is A similar to B ?

2. a) Find the distance from e_1 to $W = \text{Span} \left\{ \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix} \right\}$.

b) Find the least squares solution \hat{x} to $Ax = e_1$, where $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ -1 & 1 \end{pmatrix}$.

3. Let $A = \begin{pmatrix} 1 & 6 & 4 \\ -1 & -2 & 20 \\ 1 & 2 & -14 \\ 1 & 6 & 10 \end{pmatrix}$.

a) Find an orthogonal basis for $\text{Col}A$.

b) Find an orthonormal basis for $\text{Col}A$.

c) Find a QR decomposition for A .

4. Consider the four points $(0, 0)$, $(1, 8)$, $(3, 8)$, and $(4, 20)$.

a) Find the best fit line $y = Ax + B$ through these points.

b) Find the best fit parabola $y = Ax^2 + Bx + C$ through these points.

c) Find the best fit cubic $y = Ax^3 + Bx^2 + Cx + D$ through these points.