Activity 1.1 - Answer Key

Problem 1

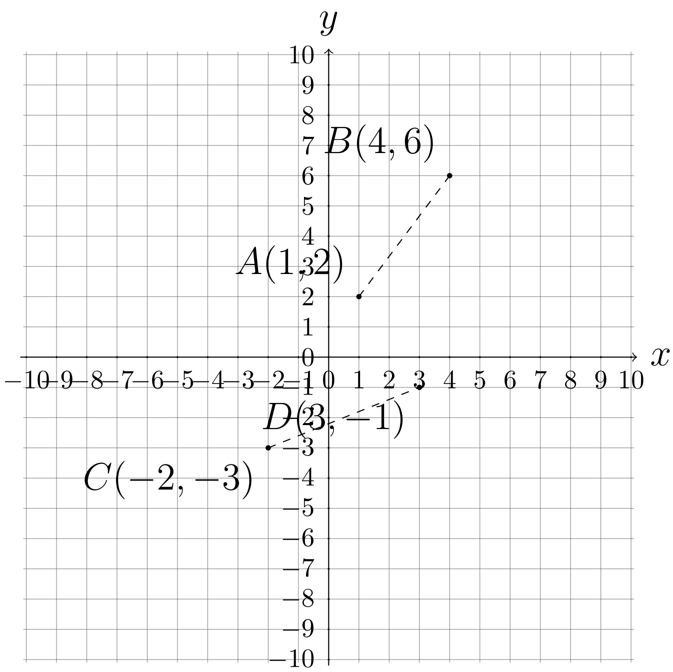
Consider the following points on the coordinate plane:

- Point A at coordinates $(1, 2)$
- Point B at coordinates $(4, 6)$
- Point C at coordinates $(-2, -3)$
- Point D at coordinates $(3, -1)$

Tasks:

1. Plot the points A, B, C, and D on a coordinate plane.
2. Find the distance between points A and B.
3. Find the distance between points C and D.

Graph:



To find the distance between two points (x_1, y_1) and (x_2, y_2) on a coordinate plane, use the distance formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Calculate the distance between points A and B:

$$AB = \sqrt{(4-1)^2 + (6-2)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

Calculate the distance between points C and D:

$$CD = \sqrt{(3+2)^2 + (-1+3)^2} = \sqrt{5^2 + 2^2} = \sqrt{25 + 4} = \sqrt{29}$$

Problem 2

Find the midpoint between $F = (-2, 5)$ and $G = (4, -7)$.

The formula for the midpoint between two points (x_1, y_1) and (x_2, y_2) is:

$$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Substitute the given points $F = (-2, 5)$ and $G = (4, -7)$ into the midpoint formula:

$$M = \left(\frac{-2 + 4}{2}, \frac{5 + (-7)}{2} \right)$$

Simplify the expressions inside the parentheses:

$$M = \left(\frac{2}{2}, \frac{-2}{2} \right)$$

Therefore, the midpoint between the points $F = (-2, 5)$ and $G = (4, -7)$ is:

$$M = (1, -1)$$

Problem 3

Graph the equation $y = 3x + 2$ (make sure to include all points from $x = -3$ to $x = 3$ and show your calculations of each point.

1. Calculate the y-values for x ranging from -3 to 3:

   - For $x = -3$:

   $$y = 3(-3) + 2 = -9 + 2 = -7$$

   - For $x = -2$:

   $$y = 3(-2) + 2 = -6 + 2 = -4$$

   - For $x = -1$:

   $$y = 3(-1) + 2 = -3 + 2 = -1$$

   - For $x = 0$:

   $$y = 3(0) + 2 = 0 + 2 = 2$$

   - For $x = 1$:

   $$y = 3(1) + 2 = 3 + 2 = 5$$

   - For $x = 2$:

   $$y = 3(2) + 2 = 6 + 2 = 8$$

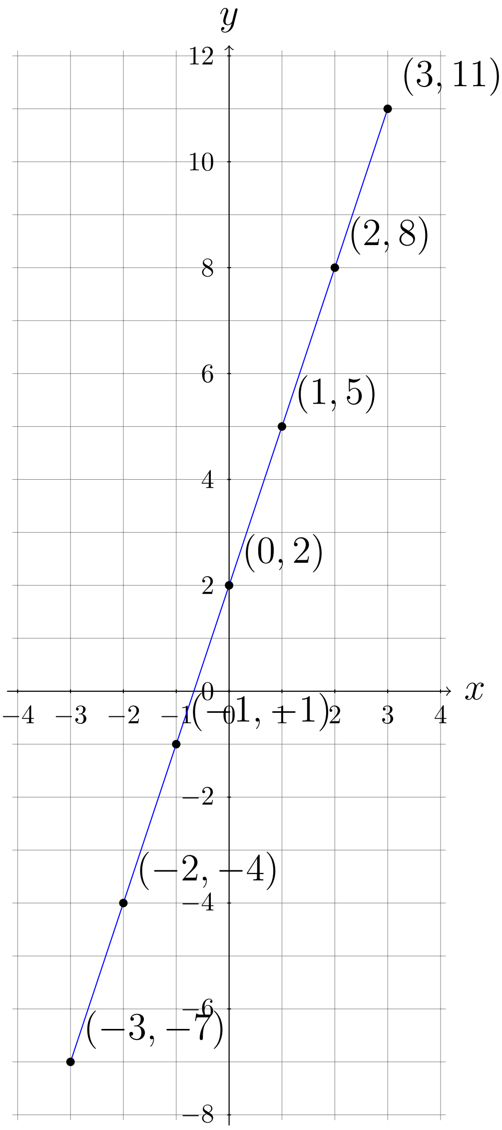
   - For $x = 3$:

   $$y = 3(3) + 2 = 9 + 2 = 11$$

2. Plot the points $(-3, -7)$, $(-2, -4)$, $(-1, -1)$, $(0, 2)$, $(1, 5)$, $(2, 8)$, and $(3, 11)$ on the coordinate plane.

3. Draw a line through these points to represent the equation $y = 3x+2$.

Graph:



Problem 4

Find the x and y intercepts of $y = x^2 - 5x + 3$

The y-intercept is the point where the graph intersects the y-axis. This occurs when $x = 0$.

Substitute $x = 0$ into the equation $y = x^2 - 5x + 3$:

$$y = (0)^2 - 5(0) + 3 = 3$$

Therefore, the y-intercept is:

$$(0, 3)$$

The x-intercepts are the points where the graph intersects the x-axis. This occurs when $y = 0$.

Set the equation equal to zero and solve for x:

$$x^2 - 5x + 3 = 0$$

Use the quadratic formula $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$, where $a = 1$, $b = -5$, and $c = 3$:

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4 \cdot 1 \cdot 3}}{2 \cdot 1}$$

Simplify inside the square root:

$$x = \frac{5 \pm \sqrt{25 - 12}}{2}$$

$$x = \frac{5 \pm \sqrt{13}}{2}$$

Therefore, the x-intercepts are:

$$x = \frac{5 + \sqrt{13}}{2} \quad \text{and} \quad x = \frac{5 - \sqrt{13}}{2}$$

So, the x-intercepts are:

$$\left(\frac{5 + \sqrt{13}}{2}, 0\right) \quad \text{and} \quad \left(\frac{5 - \sqrt{13}}{2}, 0\right)$$

Problem 5

Find the x and y intercepts of $y = \dfrac{6x + 12}{4 - 3x}$

The y-intercept is the point where the graph intersects the y-axis. This occurs when $x = 0$.

Substitute $x = 0$ into the equation $y = \frac{6x+12}{4-3x}$:

$$y = \frac{6(0) + 12}{4 - 3(0)} = \frac{12}{4} = 3$$

Therefore, the y-intercept is:

$$(0, 3)$$

The x-intercepts are the points where the graph intersects the x-axis. This occurs when $y = 0$.

Set the equation equal to zero and solve for x:

$$0 = \frac{6x + 12}{4 - 3x}$$

For the fraction to be zero, the numerator must be zero. Therefore, set the numerator equal to zero and solve for x:

$$6x + 12 = 0$$

Solve for x:

$$6x = -12$$

$$x = -2$$

Therefore, the x-intercept is:

$$(-2, 0)$$

Problem 6

Test the symmetry with the x-axis, y-axis, and origin: $y = x^2 + x$.

Given the function $y = x^2 + x$, test its symmetry with respect to the x-axis, y-axis, and the origin.

Symmetry with respect to the x-axis:

1. To test for symmetry with respect to the x-axis, replace y with $-y$ in the equation and see if the resulting equation is equivalent to the original equation.

$$y = x^2 + x \implies -y = x^2 + x$$

2. Solve for y:

$$y = -(x^2 + x) = -x^2 - x$$

3. The resulting equation $y = -x^2 - x$ is not equivalent to the original equation $y = x^2 + x$, so the function is not symmetric with respect to the x-axis.

Symmetry with respect to the y-axis:

1. To test for symmetry with respect to the y-axis, replace x with $-x$ in the equation and see if the resulting equation is equivalent to the original equation.

$$y = x^2 + x \implies y = (-x)^2 + (-x)$$

2. Simplify the resulting equation:

$$y = x^2 - x$$

3. The resulting equation $y = x^2 - x$ is not equivalent to the original equation $y = x^2 + x$, so the function is not symmetric with respect to the y-axis.

Symmetry with respect to the origin:

1. To test for symmetry with respect to the origin, replace x with $-x$ and y with $-y$ in the equation and see if the resulting equation is equivalent to the original equation.

$$y = x^2 + x \implies -y = (-x)^2 + (-x)$$

2. Simplify the resulting equation:

$$-y = x^2 - x$$

3. Solve for y:

$$y = -x^2 + x$$

4. The resulting equation $y = -x^2 + x$ is not equivalent to the original equation $y = x^2 + x$, so the function is not symmetric with respect to the origin.

Therefore, the function $y = x^2 + x$ is not symmetric with respect to the x-axis, y-axis, or the origin.

Problem 7

Given the center of a circle at the point $(h, k) = (2, -3)$ and a point that lies on the circle $(x_1, y_1) = (5, 1)$, find the standard form of the equation of the circle.

1. The standard form of the equation of a circle with center (h, k) and radius r is:

$$(x - h)^2 + (y - k)^2 = r^2$$

2. Calculate the radius r using the distance formula between the center (h, k) and the point (x_1, y_1):

$$r = \sqrt{(x_1 - h)^2 + (y_1 - k)^2}$$

Substituting the given points $(2, -3)$ and $(5, 1)$:

$$r = \sqrt{(5 - 2)^2 + (1 + 3)^2} = \sqrt{3^2 + 4^2} = \sqrt{9 + 16} = \sqrt{25} = 5$$

3. Substitute h, k, and r into the standard form equation:

$$(x-2)^2 + (y+3)^2 = 5^2$$

$$(x-2)^2 + (y+3)^2 = 25$$

4. Therefore, the standard form of the equation of the circle is:

$$(x-2)^2 + (y+3)^2 = 25$$

Problem 8

Find the center and radius of the circle: $x^2 + y^2 - 8x + 6y - 11 = 0$

Rewrite the equation in standard form by completing the square for the x and y terms.

1. Group the x terms and the y terms:

$$x^2 - 8x + y^2 + 6y = 11$$

2. Complete the square for the x and y terms:

$$x^2 - 8x + \left(\frac{-8}{2}\right)^2 + y^2 + 6y + \left(\frac{6}{2}\right)^2 = 11 + \left(\frac{-8}{2}\right)^2 + \left(\frac{6}{2}\right)^2$$

3. Simplify:

$$x^2 - 8x + 16 + y^2 + 6y + 9 = 11 + 16 + 9$$

4. Write in standard form:

$$(x-4)^2 + (y+3)^2 = 36$$

5. The standard form of a circle's equation is:

$$(x-h)^2 + (y-k)^2 = r^2$$

   Comparing this with our equation, we get:

$$h = 4, \quad k = -3, \quad r^2 = 36 \implies r = 6$$

6. Therefore, the center and radius of the circle are:

$$\text{Center: } (4, -3)$$

$$\text{Radius: } 6$$

Problem 9

True or False: If $(-2, 4)$ is a point on a graph that is symmetric with respect to the x-axis, then the point $(2, 4)$ is also on the graph.

False.

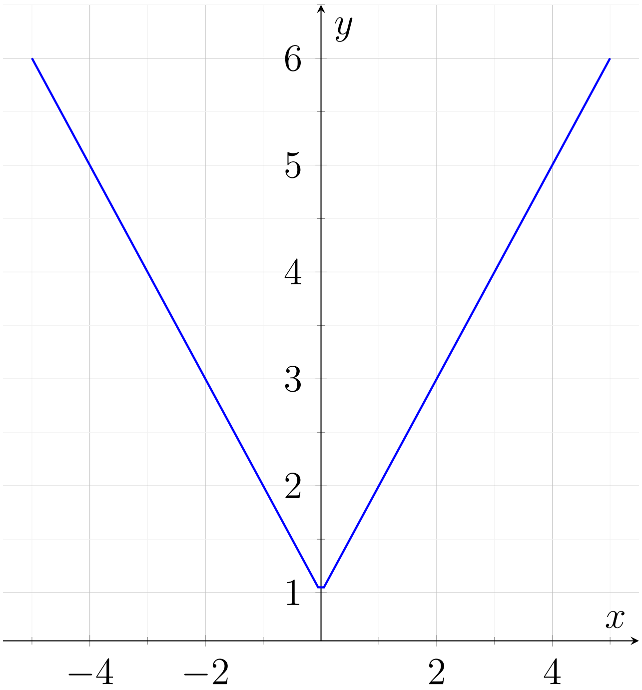
Symmetry with respect to the x-axis means that for every point (x, y) on the graph, the point $(x, -y)$ is also on the graph. Therefore, if $(-2, 4)$ is on the graph, then the point $(-2, -4)$ must also be on the graph, not $(2, 4)$.

If the graph were symmetric with respect to the y-axis, then the point $(2, 4)$ would be on the graph given that $(-2, 4)$ is on the graph.

Activity 1.1 - Answer Key

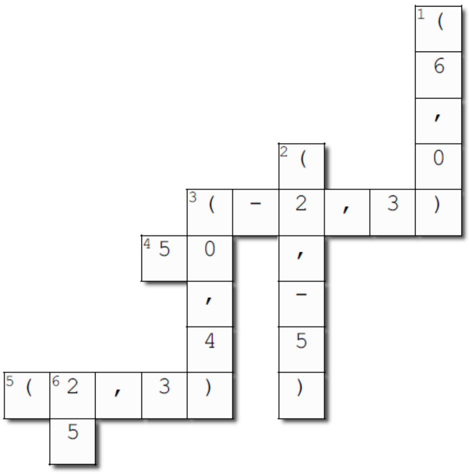
Problem 10

Graph: $y = |x| + 1$



Crossword Puzzle

Did you complete the crossword puzzle?



Across

- **3.** Find the center of: x^2 + y^2 + 4x - 6y + 1 = 0 (**(-2,3)**)
- **4.** Find the distance between (30,0) and (0,40) (**50**)
- **5.** Find the midpoint of (1,4) and (3,2). (**(2,3)**)

Down

- **1.** Find the x-intercept of: 2x + 3y = 12. Write your answer as a coordinate point. (**(6,0)**)
- **2.** Find the center of: (x - 2)^2 + (y + 5)^2 = 4 (**(2,-5)**)
- **3.** Find the y-intercept of: 2x + 3y = 12. Write your answer as a coordinate point. (**(0,4)**)
- **6.** Find the radius of: (x + 4)^2 + (y - 6)^2 = 625 (**25**)