Problem 1

Find the slope: $(2, 5)$ and $(3, -7)$

The formula for the slope m between two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the given points $(x_1, y_1) = (2, 5)$ and $(x_2, y_2) = (3, -7)$:

$$m = \frac{-7 - 5}{3 - 2}$$

Simplify the numerator and the denominator:

$$m = \frac{-12}{1}$$

So, the slope is:

$$m = -12$$

Problem 2

Find the equation of a line passing through $(-2, -4)$ and has a slope of 4.

The equation of a line in point-slope form is given by (or alternatively you may use $y = mx + b$ not shown below):

$$y - y_1 = m(x - x_1)$$

Substituting the given point $(x_1, y_1) = (-2, -4)$ and the slope $m = 4$:

$$y - (-4) = 4(x - (-2))$$

Simplify the equation:

$$y + 4 = 4(x + 2)$$

Distribute the slope on the right side:

$$y + 4 = 4x + 8$$

Subtract 4 from both sides to solve for y:

$$y = 4x + 4$$

So, the equation of the line is:

$$y = 4x + 4$$

Problem 3

Find the equation of a line passing through $(-3, -2)$ and $(3, 6)$.

First, find the slope m using the formula:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the given points $(x_1, y_1) = (-3, -2)$ and $(x_2, y_2) = (3, 6)$:

$$m = \frac{6 - (-2)}{3 - (-3)}$$

Simplify the numerator and the denominator:

$$m = \frac{6 + 2}{3 + 3} = \frac{8}{6} = \frac{4}{3}$$

Now that we have the slope, use the point-slope form of the equation of a line (or alternatively you may use $y = mx + b$ not shown below):

$$y - y_1 = m(x - x_1)$$

Substituting the slope $m = \frac{4}{3}$ and one of the points, say $(-3, -2)$:

$$y - (-2) = \frac{4}{3}(x - (-3))$$

Simplify the equation:

$$y + 2 = \frac{4}{3}(x + 3)$$

Distribute the slope on the right side:

$$y + 2 = \frac{4}{3}x + \frac{4}{3} \cdot 3$$

$$y + 2 = \frac{4}{3}x + 4$$

Subtract 2 from both sides to solve for y:

$$y = \frac{4}{3}x + 4 - 2$$

$$y = \frac{4}{3}x + 2$$

So, the equation of the line is:

$$y = \frac{4}{3}x + 2$$

Problem 4

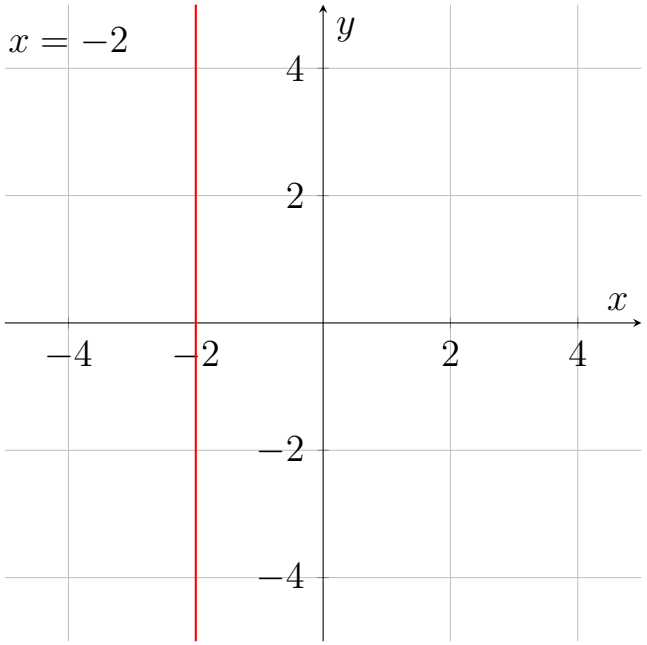
Graph the following and state each of their slopes:

1. $x = -2$
2. $y = 2x + 2$
3. $y = -2$

Line 1: $x = -2$

This is a vertical line passing through $x = -2$.

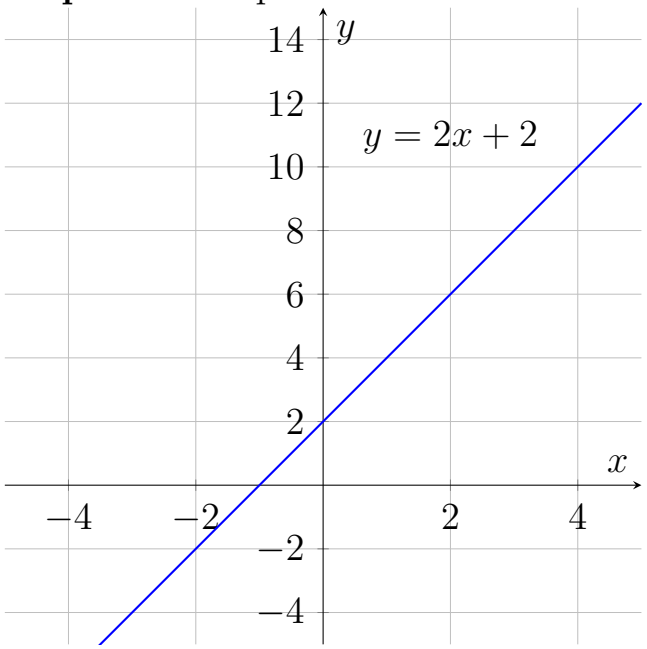
Slope: The slope of a vertical line is undefined.



Line 2: $y = 2x + 2$

This is a linear equation in slope-intercept form $y = mx + b$, where $m = 2$ and $b = 2$.

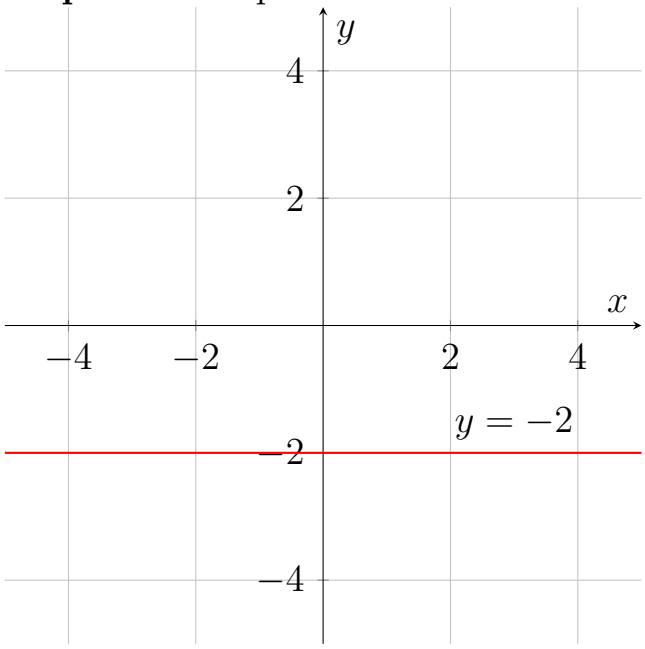
Slope: The slope of the line is 2.



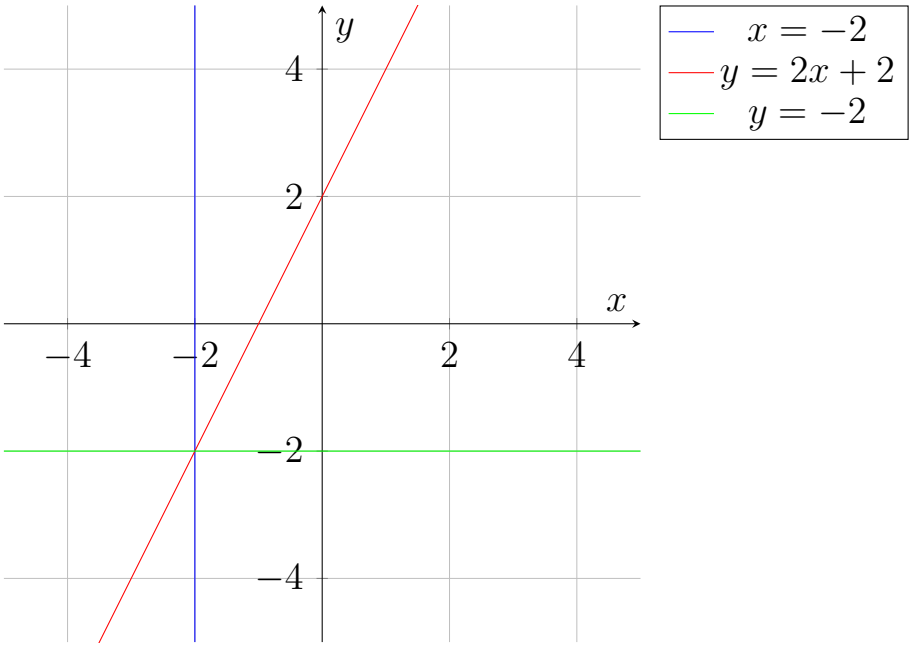
Line 3: $y = -2$

This is a horizontal line passing through $y = -2$.

Slope: The slope of a horizontal line is 0.



All 3 Graphs on One Plot



Problem 5

Determine if the lines are parallel, perpendicular, or neither: $8x+3y = 7$ and $7x - 5y = 0$.

First, we need to find the slopes of the lines by converting each equation to slope-intercept form $y = mx + b$.

Line 1: $8x + 3y = 7$

Solve for y:

$$3y = -8x + 7$$
$$y = -\frac{8}{3}x + \frac{7}{3}$$

The slope of the first line, m_1, is:

$$m_1 = -\frac{8}{3}$$

Line 2: $7x - 5y = 0$

Solve for y:

$$5y = 7x$$
$$y = \frac{7}{5}x$$

The slope of the second line, m_2, is:

$$m_2 = \frac{7}{5}$$

Comparing the Slopes

Two lines are parallel if their slopes are equal. Two lines are perpendicular if the product of their slopes is -1.

Are the Lines Parallel?

Check if $m_1 = m_2$:

$$-\frac{8}{3} \neq \frac{7}{5}$$

The slopes are not equal, so the lines are not parallel.

Are the Lines Perpendicular?

Check if $m_1 \cdot m_2 = -1$:

$$-\frac{8}{3} \cdot \frac{7}{5} = -\frac{56}{15} \neq -1$$

The product of the slopes is not -1, so the lines are not perpendicular.

Conclusion

The lines are neither parallel nor perpendicular.

Problem 6

Find the equation of a line that passes through $(3, 7)$ and is parallel to the line $6x + 3y = 36$.

Step 1: Determine the Slope of the Given Line

First, convert the given line equation to slope-intercept form $y = mx+b$.

Starting with:

$$6x + 3y = 36$$

Solve for y:

$$3y = -6x + 36$$

$$y = -2x + 12$$

The slope of the given line, m, is:

$$m = -2$$

Step 2: Use the Slope for the Parallel Line

Since parallel lines have the same slope, the slope of the line we need to find is also:

$$m = -2$$

Step 3: Use the Point-Slope Form to Find the Equation (alternatively you can use slope-intercept not shown below)

The point-slope form of a line equation is:

$$y - y_1 = m(x - x_1)$$

Substitute the given point $(x_1, y_1) = (3, 7)$ and the slope $m = -2$:

$$y - 7 = -2(x - 3)$$

Simplify the equation:

$$y - 7 = -2x + 6$$

$$y = -2x + 6 + 7$$

$$y = -2x + 13$$

Conclusion

The equation of the line that passes through $(3, 7)$ and is parallel to $6x + 3y = 36$ is:

$$y = -2x + 13$$

Problem 7

Find the equation of a line that passes through the point $(8, 2)$ and is perpendicular to the line $2x + 3y = 18$.

Step 1: Determine the Slope of the Given Line

First, convert the given line equation to slope-intercept form $y = mx+b$.

Starting with:

$$2x + 3y = 18$$

Solve for y:

$$3y = -2x + 18$$

$$y = -\frac{2}{3}x + 6$$

The slope of the given line, m, is:

$$m = -\frac{2}{3}$$

Step 2: Find the Slope of the Perpendicular Line

The slope of a line perpendicular to another is the negative reciprocal of the original slope.

The slope of the line we need to find, m', is:

$$m' = -\frac{1}{m} = -\frac{1}{-\frac{2}{3}} = \frac{3}{2}$$

(Hint: Flip the fraction, flip the sign.)

Step 3: Use the Point-Slope Form to Find the Equation (alternatively you can use slope-intercept not shown below)

The point-slope form of a line equation is:

$$y - y_1 = m'(x - x_1)$$

Substitute the given point $(x_1, y_1) = (8, 2)$ and the slope $m' = \frac{3}{2}$:

$$y - 2 = \frac{3}{2}(x - 8)$$

Simplify the equation:

$$y - 2 = \frac{3}{2}x - \frac{3}{2} \cdot 8$$

$$y - 2 = \frac{3}{2}x - 12$$

Add 2 to both sides to solve for y:

$$y = \frac{3}{2}x - 12 + 2$$

$$y = \frac{3}{2}x - 10$$

Conclusion

The equation of the line that passes through $(8, 2)$ and is perpendicular to $2x + 3y = 18$ is:

$$y = \frac{3}{2}x - 10$$

Problem 8

Find the equation of a line that passes through the point $(3, 5)$ and is perpendicular to the line containing $(1, 8)$ and $(5, 20)$.

Step 1: Determine the Slope of the Line Containing the Given Points

The slope m of a line passing through two points (x_1, y_1) and (x_2, y_2) is given by:

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Substituting the given points $(x_1, y_1) = (1, 8)$ and $(x_2, y_2) = (5, 20)$:

$$m = \frac{20 - 8}{5 - 1} = \frac{12}{4} = 3$$

The slope of the line passing through $(1, 8)$ and $(5, 20)$ is:

$$m = 3$$

Step 2: Find the Slope of the Perpendicular Line

The slope of a line perpendicular to another is the negative reciprocal of the original slope.

The slope of the line we need to find, m', is:

$$m' = -\frac{1}{m} = -\frac{1}{3}$$

(Hint: Flip the fraction, flip the sign.)

Step 3: Use the Point-Slope Form to Find the Equation (alternatively you can use slope-intercept not shown below)

The point-slope form of a line equation is:

$$y - y_1 = m'(x - x_1)$$

Substitute the given point $(x_1, y_1) = (3, 5)$ and the slope $m' = -\frac{1}{3}$:

$$y - 5 = -\frac{1}{3}(x - 3)$$

Simplify the equation:

$$y - 5 = -\frac{1}{3}x + \frac{1}{3} \cdot 3$$

$$y - 5 = -\frac{1}{3}x + 1$$

Add 5 to both sides to solve for y:

$$y = -\frac{1}{3}x + 1 + 5$$

$$y = -\frac{1}{3}x + 6$$

Conclusion

The equation of the line that passes through $(3, 5)$ and is perpendicular to the line containing $(1, 8)$ and $(5, 20)$ is:

$$y = -\frac{1}{3}x + 6$$