Problem 1

Find the domain: $f(x) = \frac{1}{x^2-9}$

To find the domain of the function $f(x) = \frac{1}{x^2-9}$, we need to determine the values of x for which the function is defined.

Step 1: Identify the restriction on the denominator

The function $f(x)$ is undefined when the denominator is zero:

$$x^2 - 9 = 0$$

Step 2: Solve the equation for x

Solve for x to find the values that make the denominator zero:

$$x^2 - 9 = 0 \implies (x-3)(x+3) = 0$$

Thus, the solutions are:

$$x = 3 \quad \text{or} \quad x = -3$$

Step 3: State the domain

The domain of $f(x)$ is all real numbers except where the denominator is zero. In interval notation it is:

$$(-\infty, -3) \cup (-3, 3) \cup (3, \infty)$$

Problem 2

Find the domain: $f(x) = \sqrt{x-8}$

To find the domain of the function $f(x) = \sqrt{x-8}$, we need to determine the values of x for which the function is defined.

Step 1: Identify the restriction on the radicand

The function $f(x)$ is defined when the expression inside the square root is non-negative:

$$x - 8 \geq 0$$

Step 2: Solve the inequality for x

Solve the inequality to find the values that make the radicand non-negative:

$$x - 8 \geq 0 \implies x \geq 8$$

Step 3: State the domain

The domain of $f(x)$ is all real numbers x such that x is greater than or equal to 8. In interval notation it is:

$$[8, \infty)$$

Problem 3
Find the domain: $g(x) = \dfrac{3x}{\sqrt{3 - x}}$

To find the domain, we need to determine the values of x for which the function is defined. This involves two main considerations:

1. The denominator $\sqrt{3 - x}$ must not be zero. 2. The expression inside the square root, $3 - x$, must be non-negative (since the square root function is only defined for non-negative arguments).

Step 1: Ensure the Denominator is Not Zero

The denominator $\sqrt{3-x}$ must not be zero:

$$\sqrt{3-x} \neq 0$$

Squaring both sides:

$$3 - x \neq 0$$

$$x \neq 3$$

Step 2: Ensure the Expression Inside the Square Root is Non-Negative

The expression inside the square root must be non-negative:

$$3 - x \geq 0$$

Solving for x:

$$x \leq 3$$

Step 3: Combine the Conditions

Combining the conditions from Step 1 and Step 2: $x \leq 3$ and $x \neq 3$ Therefore, the domain of $g(x)$ is all real numbers x such that $x < 3$.

Conclusion

The denominator can't be zero and you can't take the square root of a negative number, so $3 - x > 0$. The domain of the function in interval notation is:

$$(-\infty, 3)$$

Problem 4

Given $f(x) = x^3 + 2x^2 - 3x + 1$ and $g(x) = \frac{16}{\sqrt{x^2}}$, find $f(g(4))$

Step 1: Evaluate $g(4)$

First, we need to find $g(4)$:

$$g(x) = \frac{16}{\sqrt{x^2}}$$

Since $\sqrt{x^2} = |x|$:

$$g(x) = \frac{16}{|x|}$$

Substituting $x = 4$:

$$g(4) = \frac{16}{|4|} = \frac{16}{4} = 4$$

Step 2: Evaluate $f(g(4))$

Next, we need to find $f(g(4))$. Since $g(4) = 4$, we need to find $f(4)$:

$$f(x) = x^3 + 2x^2 - 3x + 1$$

Substituting $x = 4$:

$$f(4) = 4^3 + 2(4^2) - 3(4) + 1$$

Calculate each term:

$$4^3 = 64$$

$$2(4^2) = 2 \cdot 16 = 32$$

$$-3(4) = -12$$

Add the terms together:

$$f(4) = 64 + 32 - 12 + 1$$

$$f(4) = 85$$

Conclusion

Therefore, $f(g(4))$ is:

$$f(g(4)) = 85$$

Problem 5

Given the function $f(x) = x^2 + 3$ with domain $x = [-2, 3]$, what is the range of the function in this interval?

Step 1: Evaluate the Function at the Endpoints of the Domain

First, we need to evaluate the function at the endpoints of the given domain, $x = -2$ and $x = 3$.

$$f(-2) = (-2)^2 + 3 = 4 + 3 = 7$$
$$f(3) = 3^2 + 3 = 9 + 3 = 12$$

Step 2: Determine the Behavior of the Function Within the Interval

The function $f(x) = x^2 + 3$ is a quadratic function, which opens upwards (since the coefficient of x^2 is positive). This means that the minimum value of $f(x)$ within the interval will occur at the smallest value of x in the domain, and the maximum value will occur at the largest value of x in the domain.

Step 3: Evaluate the Function at Critical Points

Since $f(x) = x^2 + 3$ is a parabola opening upwards, its vertex is at the lowest point. However, we need to check the interval $[-2, 3]$. The function is increasing on $[0, 3]$ and decreasing on $[-2, 0]$, with its minimum at $x = 0$.

Evaluate the function at $x = 0$:

$$f(0) = 0^2 + 3 = 3$$

Step 4: Determine the Range of the Function

Given the function's values at the endpoints and the vertex:

$$f(-2) = 7, \quad f(0) = 3, \quad f(3) = 12$$

Since $f(x)$ is continuous and increases monotonically within $[0, 3]$ and decreases monotonically within $[-2, 0]$, the minimum value within the interval is 3 and the maximum value is 12.

Conclusion

Therefore, the range of the function $f(x) = x^2 + 3$ for $x \in [-2, 3]$ is:

$$[3, 12]$$

Problem 6

Find the average rate of change of the function as x changes from a to b given $f(x) = 2x^2 + 2$, $a = 1$, $b = 5$

Step 1: Evaluate the Function at the Endpoints

First, we need to find the values of the function at $x = a$ and $x = b$.

Evaluate $f(a)$:

$$f(1) = 2(1)^2 + 2 = 2 \cdot 1 + 2 = 4$$

Evaluate $f(b)$:

$$f(5) = 2(5)^2 + 2 = 2 \cdot 25 + 2 = 50 + 2 = 52$$

Step 2: Use the Average Rate of Change Formula

The average rate of change of a function $f(x)$ over the interval $[a, b]$ is given by:

$$\frac{f(b) - f(a)}{b - a}$$

Substitute the values $f(a) = 4$, $f(b) = 52$, $a = 1$, and $b = 5$:

$$\frac{f(5) - f(1)}{5 - 1} = \frac{52 - 4}{5 - 1} = \frac{48}{4} = 12$$

Conclusion

Therefore, the average rate of change of the function $f(x) = 2x^2 + 2$ as x changes from $a = 1$ to $b = 5$ is:

$$\boxed{12}$$

Problem 7

Find the average rate of change of the function as x changes from a to b given $f(x) = x^2 + 2x - 4$, $a = -2$, $b = 2$

Step 1: Evaluate the Function at the Endpoints

First, we need to find the values of the function at $x = a$ and $x = b$.

Evaluate $f(a)$:

$$f(-2) = (-2)^2 + 2(-2) - 4 = 4 - 4 - 4 = -4$$

Evaluate $f(b)$:

$$f(2) = 2^2 + 2(2) - 4 = 4 + 4 - 4 = 4$$

Step 2: Use the Average Rate of Change Formula

The average rate of change of a function $f(x)$ over the interval $[a, b]$ is given by:

$$\frac{f(b) - f(a)}{b - a}$$

Substitute the values $f(a) = -4$, $f(b) = 4$, $a = -2$, and $b = 2$:

$$\frac{f(2) - f(-2)}{2 - (-2)} = \frac{4 - (-4)}{2 - (-2)} = \frac{4 + 4}{2 + 2} = \frac{8}{4} = 2$$

Conclusion

Therefore, the average rate of change of the function $f(x) = x^2 + 2x - 4$ as x changes from $a = -2$ to $b = 2$ is:

$$\boxed{2}$$

Problem 8

Find and simplify the difference quotient of: $f(x) = 4x + 6$

Step 1: Calculate $f(x + h)$

Substitute $x + h$ into the function $f(x)$:

$$f(x + h) = 4(x + h) + 6$$

Simplify the expression:

$$f(x+h) = 4x + 4h + 6$$

Step 2: Substitute into the Difference Quotient Formula

Substitute $f(x+h)$ and $f(x)$ into the difference quotient formula:

$$\frac{f(x+h) - f(x)}{h} = \frac{(4x + 4h + 6) - (4x + 6)}{h}$$

Step 3: Simplify the Expression

Simplify the numerator:

$$\frac{(4x + 4h + 6) - (4x + 6)}{h} = \frac{4x + 4h + 6 - 4x - 6}{h}$$

Combine like terms:

$$\frac{4h}{h}$$

Simplify the fraction:

$$4$$

Conclusion

The simplified difference quotient of the function $f(x) = 4x + 6$ is:

$$\boxed{4}$$

Problem 9

Find and simplify the difference quotient of: $f(x) = -2x^2 + 2x$

Step 1: Calculate $f(x + h)$

Substitute $x + h$ into the function $f(x)$:

$$f(x + h) = -2(x + h)^2 + 2(x + h)$$

Expand and simplify the expression:

$$f(x + h) = -2(x^2 + 2xh + h^2) + 2x + 2h$$

$$f(x + h) = -2x^2 - 4xh - 2h^2 + 2x + 2h$$

Step 2: Substitute into the Difference Quotient Formula

Substitute $f(x + h)$ and $f(x)$ into the difference quotient formula:

$$\frac{f(x + h) - f(x)}{h} = \frac{(-2x^2 - 4xh - 2h^2 + 2x + 2h) - (-2x^2 + 2x)}{h}$$

Step 3: Simplify the Expression

Simplify the numerator:

$$\frac{(-2x^2 - 4xh - 2h^2 + 2x + 2h) - (-2x^2 + 2x)}{h}$$

$$= \frac{-2x^2 - 4xh - 2h^2 + 2x + 2h + 2x^2 - 2x}{h}$$

Combine like terms:

$$\frac{-4xh - 2h^2 + 2h}{h}$$

Factor out h from the numerator:

$$\frac{h(-4x - 2h + 2)}{h}$$

Cancel h in the numerator and denominator:

$$-4x - 2h + 2$$

Conclusion

The simplified difference quotient of the function $f(x) = -2x^2 + 2x$ is:

$$\boxed{-4x + 2 - 2h}$$