Problem 1-4

Given $f(x) = x - 4$ and $g(x) = x^2 - 2x - 8$

1. $(f + g)(x)$

2. $(f - g)(x)$

3. $(f \cdot g)(x)$

4. $(\frac{f}{g})(x)$

1. $(f + g)(x)$

   To find $(f + g)(x)$, we add the functions $f(x)$ and $g(x)$:

   $$(f + g)(x) = f(x) + g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f + g)(x) = (x - 4) + (x^2 - 2x - 8)$$

   Combine like terms:

   $$(f + g)(x) = x^2 - x - 12$$

2. $(f - g)(x)$

   To find $(f - g)(x)$, we subtract the function $g(x)$ from $f(x)$:

   $$(f - g)(x) = f(x) - g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f - g)(x) = (x - 4) - (x^2 - 2x - 8)$$

   Distribute the negative sign and combine like terms:

   $$(f - g)(x) = x - 4 - x^2 + 2x + 8$$

$$(f-g)(x) = -x^2 + 3x + 4$$

3. $(f \cdot g)(x)$

   To find $(f \cdot g)(x)$, we multiply the functions $f(x)$ and $g(x)$:

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

   Substitute $f(x)$ and $g(x)$:

$$(f \cdot g)(x) = (x-4)(x^2 - 2x - 8)$$

   Expand and simplify:

$$(f \cdot g)(x) = x(x^2 - 2x - 8) - 4(x^2 - 2x - 8)$$

$$(f \cdot g)(x) = x^3 - 2x^2 - 8x - 4x^2 + 8x + 32$$

   Combine like terms:

$$(f \cdot g)(x) = x^3 - 6x^2 + 32$$

4. $\left(\frac{f}{g}\right)(x)$

   To find $\left(\frac{f}{g}\right)(x)$, we divide the function $f(x)$ by $g(x)$:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

   Substitute $f(x)$ and $g(x)$:

$$\left(\frac{f}{g}\right)(x) = \frac{x-4}{x^2 - 2x - 8}$$

   The simplified form is:

$$\left(\frac{f}{g}\right)(x) = \frac{x-4}{(x-4)(x+2)}$$

$$\left(\frac{f}{g}\right)(x) = \frac{1}{x+2}, \quad \text{for} \quad x \neq 4$$

Problem 5-8

Given $f(x) = 3x$ and $g(x) = -x$

5. $(f + g)(-1)$
6. $(f - g)(0)$
7. $(f \cdot g)(2)$
8. $(\frac{f}{g})(1)$

5. $(f + g)(-1)$

   To find $(f + g)(-1)$, we first find $(f + g)(x)$:

   $$(f + g)(x) = f(x) + g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f + g)(x) = 3x + (-x) = 2x$$

   Now, evaluate at $x = -1$:

   $$(f + g)(-1) = 2(-1) = -2$$

6. $(f - g)(0)$

   To find $(f - g)(0)$, we first find $(f - g)(x)$:

   $$(f - g)(x) = f(x) - g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f - g)(x) = 3x - (-x) = 3x + x = 4x$$

   Now, evaluate at $x = 0$:

   $$(f - g)(0) = 4(0) = 0$$

7. $(f \cdot g)(2)$

   To find $(f \cdot g)(2)$, we first find $(f \cdot g)(x)$:

$$(f \cdot g)(x) = f(x) \cdot g(x)$$

   Substitute $f(x)$ and $g(x)$:

$$(f \cdot g)(x) = (3x)(-x) = -3x^2$$

   Now, evaluate at $x = 2$:

$$(f \cdot g)(2) = -3(2)^2 = -3(4) = -12$$

8. $\left(\frac{f}{g}\right)(1)$

   To find $\left(\frac{f}{g}\right)(1)$, we first find $\left(\frac{f}{g}\right)(x)$:

$$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

   Substitute $f(x)$ and $g(x)$:

$$\left(\frac{f}{g}\right)(x) = \frac{3x}{-x} = -3$$

   Now, evaluate at $x = 1$:

$$\left(\frac{f}{g}\right)(1) = -3$$

Problem 9-12

Given $f(x) = 4x + 2$ and $g(x) = -x^2 + 2$

9. $(f+g)(1)$

10. $(f-g)(2)$

11. $(f \cdot g)(-3)$

12. $(\frac{f}{g})(1)$

9. $(f+g)(1)$

   To find $(f+g)(1)$, we first find $(f+g)(x)$:

   $$(f+g)(x) = f(x) + g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f+g)(x) = (4x+2) + (-x^2+2) = -x^2 + 4x + 4$$

   Now, evaluate at $x = 1$:

   $$(f+g)(1) = -(1)^2 + 4(1) + 4 = -1 + 4 + 4 = 7$$

10. $(f-g)(2)$

   To find $(f-g)(2)$, we first find $(f-g)(x)$:

   $$(f-g)(x) = f(x) - g(x)$$

   Substitute $f(x)$ and $g(x)$:

   $$(f-g)(x) = (4x+2) - (-x^2+2) = 4x + 2 + x^2 - 2 = x^2 + 4x$$

   Now, evaluate at $x = 2$:

   $$(f-g)(2) = (2)^2 + 4(2) = 4 + 8 = 12$$

11. $(f \cdot g)(-3)$

    To find $(f \cdot g)(-3)$, we first find $(f \cdot g)(x)$:

    $$(f \cdot g)(x) = f(x) \cdot g(x)$$

    Substitute $f(x)$ and $g(x)$:

    $$(f \cdot g)(x) = (4x + 2)(-x^2 + 2)$$

    Expand and simplify:

    $$(f \cdot g)(x) = 4x(-x^2) + 4x(2) + 2(-x^2) + 2(2)$$

    $$(f \cdot g)(x) = -4x^3 + 8x - 2x^2 + 4$$

    Now, evaluate at $x = -3$:

    $$(f \cdot g)(-3) = -4(-3)^3 + 8(-3) - 2(-3)^2 + 4$$

    $$(f \cdot g)(-3) = -4(-27) - 24 - 2(9) + 4$$

    $$(f \cdot g)(-3) = 108 - 24 - 18 + 4 = 70$$

12. $\left(\frac{f}{g}\right)(1)$

    To find $\left(\frac{f}{g}\right)(1)$, we first find $\left(\frac{f}{g}\right)(x)$:

    $$\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

    Substitute $f(x)$ and $g(x)$:

    $$\left(\frac{f}{g}\right)(x) = \frac{4x + 2}{-x^2 + 2}$$

    Now, evaluate at $x = 1$:

    $$\left(\frac{f}{g}\right)(1) = \frac{4(1) + 2}{-(1)^2 + 2} = \frac{4 + 2}{-1 + 2} = \frac{6}{1} = 6$$

Problem 13

Given $f(x) = 2x + 1$ and $g(x) = 2x^2 - 3$, find $(f \circ g)(x)$

Step 1: Substitute $g(x)$ into $f(x)$

To find $f(g(x))$, we substitute $g(x)$ into $f(x)$:

$$f(g(x)) = f(2x^2 - 3)$$

Step 2: Evaluate $f(2x^2 - 3)$

Now we evaluate $f(2x^2 - 3)$ using the definition of $f(x)$:

$$f(x) = 2x + 1$$

So, substituting $2x^2 - 3$ for x in $f(x)$, we get:

$$f(2x^2 - 3) = 2(2x^2 - 3) + 1$$

Step 3: Simplify the Expression

Next, we simplify the expression:

$$f(2x^2 - 3) = 2 \cdot 2x^2 - 2 \cdot 3 + 1$$

$$f(2x^2 - 3) = 4x^2 - 6 + 1$$

$$f(2x^2 - 3) = 4x^2 - 5$$

Conclusion

Therefore, the composition of the functions f and g, $(f \circ g)(x)$, is:

$$\boxed{(f \circ g)(x) = 4x^2 - 5}$$

Problem 14

Given $f(x) = 2x + 1$ and $g(x) = 2x^2 - 3$, find $(g \circ f)(x)$

Step 1: Substitute $f(x)$ into $g(x)$

To find $g(f(x))$, we substitute $f(x)$ into $g(x)$:

$$g(f(x)) = g(2x + 1)$$

Step 2: Evaluate $g(2x + 1)$

Now we evaluate $g(2x + 1)$ using the definition of $g(x)$:

$$g(x) = 2x^2 - 3$$

So, substituting $2x + 1$ for x in $g(x)$, we get:

$$g(2x + 1) = 2(2x + 1)^2 - 3$$

Step 3: Simplify the Expression

Next, we simplify the expression:

$$g(2x + 1) = 2(2x + 1)^2 - 3$$

First, expand $(2x + 1)^2$:

$$(2x + 1)^2 = (2x + 1)(2x + 1) = 4x^2 + 4x + 1$$

Then, multiply by 2:

$$2(4x^2 + 4x + 1) = 8x^2 + 8x + 2$$

Finally, subtract 3:

$$8x^2 + 8x + 2 - 3 = 8x^2 + 8x - 1$$

Conclusion

Therefore, the composition of the functions g and f, $(g \circ f)(x)$, is:

$$\boxed{(g \circ f)(x) = 8x^2 + 8x - 1}$$

Problem 15

Given $f(x) = x^2 + 2$ and $g(x) = 1 - 2x$. Find the average rate of change of the composite function $f \circ g$ as x changes from $a = 1$ to $b = 2$.

Step 1: Find the Composite Function $(f \circ g)(x)$

To find $(f \circ g)(x)$, we substitute $g(x)$ into $f(x)$:

$$(f \circ g)(x) = f(g(x))$$

Substitute $g(x) = 1 - 2x$ into $f(x) = x^2 + 2$:

$$(f \circ g)(x) = f(1 - 2x) = (1 - 2x)^2 + 2$$

Step 2: Simplify the Composite Function

Simplify $(1 - 2x)^2 + 2$:

$$(1 - 2x)^2 = 1 - 4x + 4x^2$$

$$(f \circ g)(x) = 1 - 4x + 4x^2 + 2 = 4x^2 - 4x + 3$$

Step 3: Evaluate the Composite Function at $x = 1$ and $x = 2$

Evaluate $(f \circ g)(1)$:

$$(f \circ g)(1) = 4(1)^2 - 4(1) + 3 = 4 - 4 + 3 = 3$$

Evaluate $(f \circ g)(2)$:

$$(f \circ g)(2) = 4(2)^2 - 4(2) + 3 = 16 - 8 + 3 = 11$$

Step 4: Find the Average Rate of Change

The average rate of change of $(f \circ g)(x)$ from $x = 1$ to $x = 2$ is given by:

$$\text{Average Rate of Change} = \frac{(f \circ g)(b) - (f \circ g)(a)}{b - a}$$

Substitute $a = 1$, $b = 2$, $(f \circ g)(1) = 3$, and $(f \circ g)(2) = 11$:

$$\text{Average Rate of Change} = \frac{11 - 3}{2 - 1} = \frac{8}{1} = 8$$

Conclusion

Therefore, the average rate of change of the composite function $(f \circ g)(x)$ as x changes from $a = 1$ to $b = 2$ is:

$$\boxed{8}$$

Problem 16

Given $f(x) = x^2 - 3$ and $g(x) = 3x - 4$. Find the average rate of change of the composite function $g \circ f$ as x changes from $a = -3$ to $b = 4$.

Step 1: Find the Composite Function $g \circ f$

The composite function $g \circ f$ is defined as:

$$(g \circ f)(x) = g(f(x))$$

First, find $f(x)$:

$$f(x) = x^2 - 3$$

Next, substitute $f(x)$ into g:

$$(g \circ f)(x) = g(f(x)) = g(x^2 - 3)$$

Now, apply the function g to $x^2 - 3$:

$$g(x^2 - 3) = 3(x^2 - 3) - 4 = 3x^2 - 9 - 4 = 3x^2 - 13$$

So, the composite function is:

$$(g \circ f)(x) = 3x^2 - 13$$

Step 2: Find the Values of $(g \circ f)(a)$ and $(g \circ f)(b)$

Evaluate the composite function at $a = -3$ and $b = 4$:

$$(g \circ f)(-3) = 3(-3)^2 - 13 = 3(9) - 13 = 27 - 13 = 14$$

$$(g \circ f)(4) = 3(4)^2 - 13 = 3(16) - 13 = 48 - 13 = 35$$

Step 3: Calculate the Average Rate of Change

The average rate of change of the function $g \circ f$ from $x = a$ to $x = b$ is given by:

$$\frac{(g \circ f)(b) - (g \circ f)(a)}{b - a}$$

Substitute the values found in Step 2:

$$\frac{(g \circ f)(4) - (g \circ f)(-3)}{4 - (-3)} = \frac{35 - 14}{4 - (-3)} = \frac{21}{4 + 3} = \frac{21}{7} = 3$$

Conclusion

The average rate of change of the composite function $g \circ f$ as x changes from $a = -3$ to $b = 4$ is:

$$\boxed{3}$$

Problem 17

True or False: $(f \circ g)(x)$ is always equal to $(g \circ f)(x)$

False, they can be equal, but not always as it depends on what f and g are as functions.

Secret Phrase

What was the secret phrase you found?

TO HELL WITH GEORGIA