Problem 1

Fill in the blank: A number c for which $f(c) = 0$ is called a(n) _____ of the polynomial function f.

zero or x-intercept

Problem 2

What is the degree of the polynomial: $f(x) = x^4 - 3x^7 + 3x - 2$

The degree is the exponent of the highest power term when the polynomial is in general form, which is 7.

Problem 3

Describe the end behavior of: $f(x) = -x^5 + 3x^2 - x + 1$

To find the end behavior, you need to look at the degree and leading coefficient. The polynomial has an odd degree, 5, and a negative leading coefficient, -1. Hence, the end behavior is up, down.

Problem 4

Describe the end behavior of: $f(x) = -x^2(x-3)^5(x+5)^3$

To find the end behavior, you need to look at the degree and leading coefficient. The degree is $2 + 5 + 3 = 10$, which is even, and the leading coefficient is $-1(1)^5(1)^3 = -1$, which is negative, so the end behavior is down, down.

Problem 5

What is the fewest number of real zeros that a polynomial with degree $n > 0$ can have? (answer as a number, e.g. 1, 2, or 3)

A polynomial can have as few as 0 real zeros, e.g. $f(x) = x^2 + 1$

Problem 6

Find the zeros of the polynomial: $f(x) = 5x^3 - 45x$

Factor the polynomial and use the zero product rule to solve for x,

$f(x) = 5x(x^2 - 9) = 5x(x - 3)(x + 3)$

$5x = 0$ and $x - 3 = 0$ and $x + 3 = 0$

Hence the zeros are: $0, 3, -3$

Problem 7

Find the multiplicity of each zero: $f(x) = x^3(x - 5)^2(x + 8)^9$ (in order from left to right)

The multiplicities are the exponents of each zero: 3, 2, and 9

Problem 8

Find whether the graph crosses or touches each zero:

$f(x) = (x - 2)^2(x + 3)^3$ (in order from left to right)

The multiplicities of the zeros are 2, and 3, which are even and odd, respectively. Therefore, the graph **touches (multiplicity even number)** at $x = 2$ and **crosses (multiplicity odd number)** at $x = -3$

Problem 9

Without graphing, can you prove that there is a zero between $x = -1$ and $x = 1$ of $f(x) = -x^3 - x + 3$?

Using the Intermediate Value Theorem, $f(-1) = 5$ and $f(1) = 1$, which have the same sign (both result in a positive y-value). Therefore, we cannot conclude that there is a zero. There "maybe" is a zero. We would need to do more graphing steps to conclude a yes or no.

Graphing problems:

Problem 10

Graph: $f(x) = x^3(x+2)^2(x-5)^3$

Degree: $3 + 2 + 3 = 8$

Leading Coefficient: 1

End Behavior: up, up

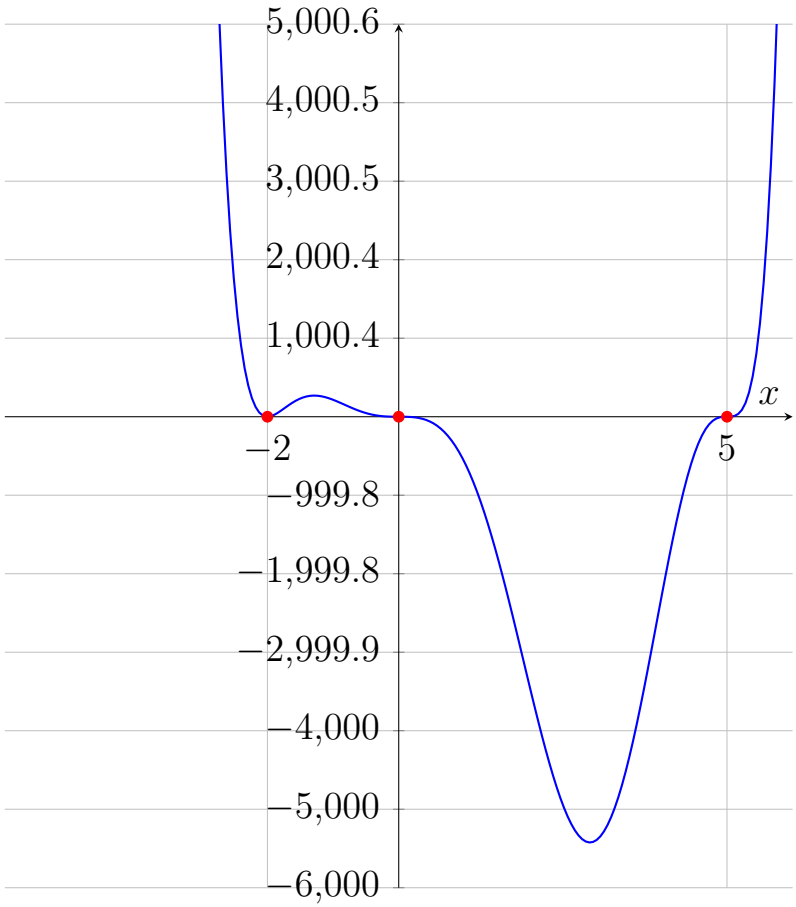
Zeros (in order left to right): 0, -2, 5

Multiplicities (in order left to right): 3, 2, and 3

Cross/Touch (in order left to right): cross, touch, cross

y-intercept: $y = 0 * 2^2 * (-5)^3 = 0$

NLT: +, +, -, +



Problem 11

Graph: $f(x) = -(x+3)^2(x-1)^3$

Degree: $2 + 3 = 5$

Leading Coefficient: -1

End Behavior: up, down

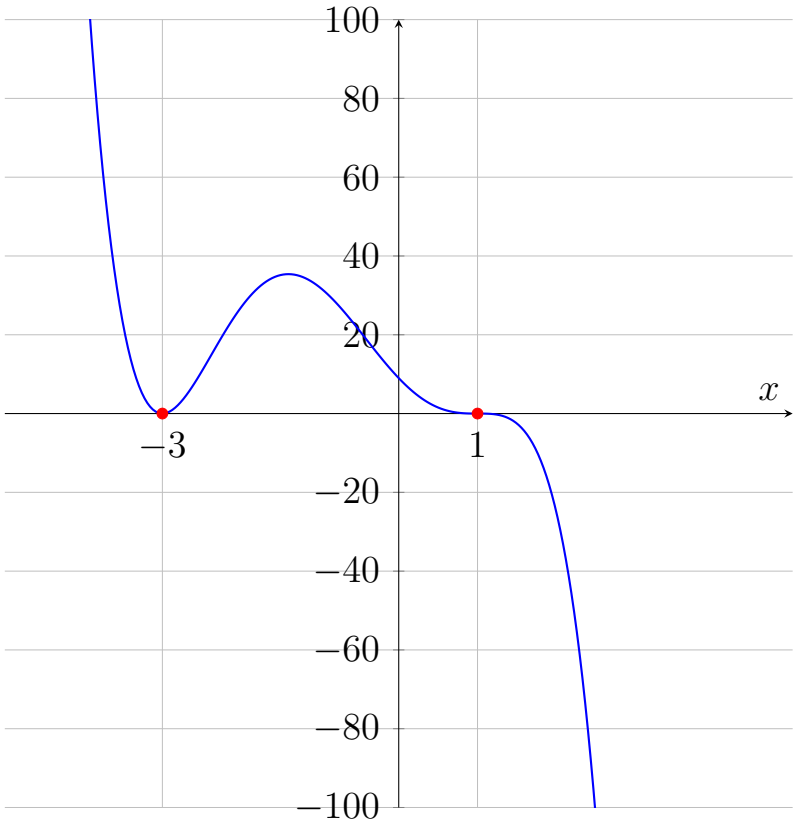
Zeros (in order left to right): $-3, 1$

Multiplicities (in order left to right): 2, 3

Cross/Touch (in order left to right): touch, cross

y-intercept is: $y = -3^2 * (-1)^3 = 9$

NLT: + + -



Problem 12

$f(x) = x(x+5)^2$

Degree: $1 + 2 = 3$

Leading Coefficient: 1

End Behavior: down, up

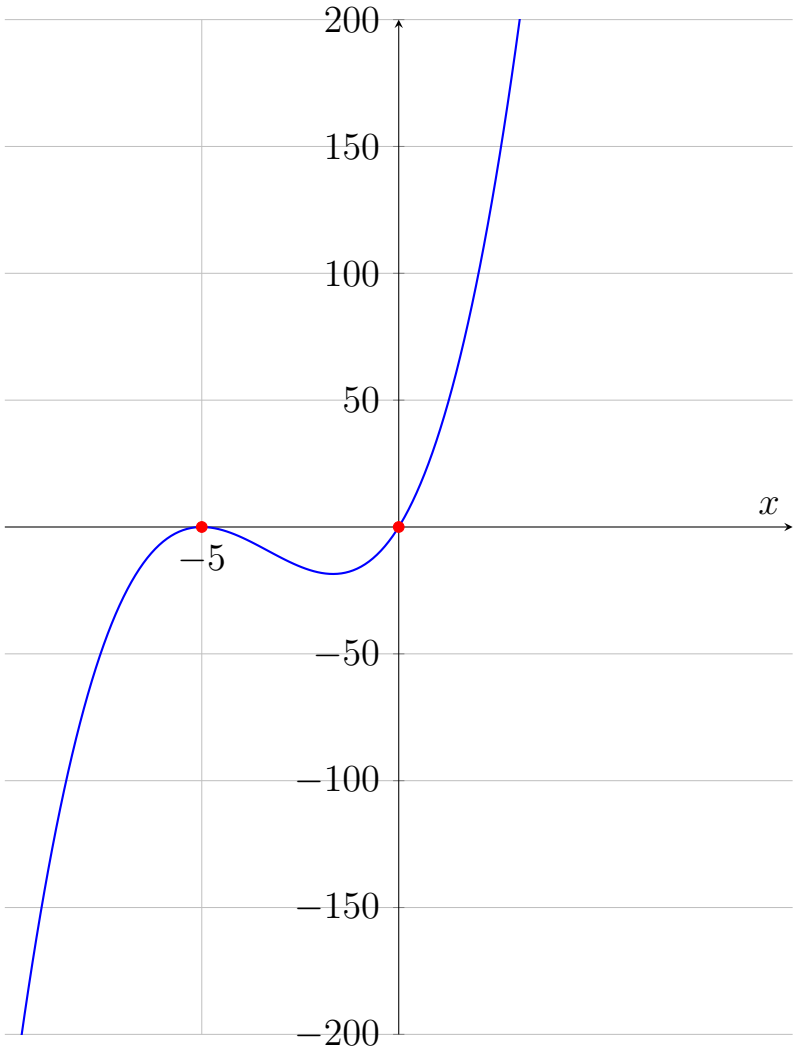
Zeros (in order left to right): $0, -5$

Multiplicities (in order left to right): 1, 2

Cross/Touch (in order left to right): cross, touch

y-intercept is: $y = 0$

NLT: - - +



Problem 13

Graph: $f(x) = -x^2(x-2)^3(x+3)$

Degree: $2+3+1=6$

Leading Coefficient: -1

End Behavior: down, down

Zeros (in order left to right): $0, 2, -3$

Multiplicities (in order left to right): 2, 3, 1

Cross/Touch (in order left to right): touch, cross, cross

y-intercept is: $y = 0$

NLT: - + + -