Problem 1

Divide using long division: $(x^4 - 2x^2 + 1) \div (x^2 - 2x + 1)$

$$
\begin{array}{rrrrrr}
 & & & x^2 & +\,2x & +\,1 \\
\cline{2-6}
x^2-2x+1 \big) & x^4 & & -\,2x^2 & & +\,1 \\
 & -\,x^4 & +\,2x^3 & -\,x^2 & & \\
\cline{2-4}
 & & 2x^3 & -\,3x^2 & & \\
 & & -\,2x^3 & +\,4x^2 & -\,2x & \\
\cline{3-5}
 & & & x^2 & -\,2x & +\,1 \\
 & & & -\,x^2 & +\,2x & -\,1 \\
\cline{4-6}
 & & & & & 0
\end{array}
$$

Therefore, the result of the division is $x^2 + 2x + 1$

Problem 2

Divide using long division: $(4x^4 + 4x^2 - x) \div (2x^2 - 1)$

$$
\begin{array}{rrrrr}
 & & 2x^2 & & +\,3 \\
\cline{2-5}
2x^2-1 \big) & 4x^4 & +\,4x^2 & -\,x & \\
 & -\,4x^4 & +\,2x^2 & & \\
\cline{2-3}
 & & 6x^2 & -\,x & \\
 & & -\,6x^2 & & +\,3 \\
\cline{3-5}
 & & & -\,x & +\,3
\end{array}
$$

Therefore, the result of the division is $2x^2 + 3 + \frac{-x+3}{2x^2-1}$

Problem 3

Divide using synthetic division: $(x^3 + x^2 - 13x + 2) \div (x - 2)$

$$\begin{array}{r|rrrr} & 1 & 1 & -13 & 2 \\ 2 & & 2 & 6 & -14 \\ \hline & 1 & 3 & -7 & -12 \end{array}$$

Therefore, the result of the division is $x^2 + 3x - 7 + \frac{-12}{x-2}$

Problem 4

Use synthetic division to factor the polynomial $x^3 + x^2 - 2$ given that $x = 1$ is a zero.

$x = 1$ being a zero implies $x - 1$ should be a factor of the polynomial. We will use synthetic division to find the quotient of $x^3 + x^2 - 2$ and $x - 1$.

$$\begin{array}{r|rrrr} & 1 & 1 & 0 & -2 \\ 1 & & 1 & 2 & 2 \\ \hline & 1 & 2 & 2 & 0 \end{array}$$

Therefore, the polynomial can be written as $(x - 1)(x^2 + 2x + 2)$. A quick check using the quadratic formula shows that $x^2 + 2x + 2$ cannot be factored further, so this is the fully factored form.

Problem 5

When can you use synthetic division versus long division?

You can use long division for any two polynomials. However, you can only use synthetic division when the divisor is of the form $x \pm c$ for some real number c.

Problem 6

Find the remainder of $(x^3 - 3x^2 + 8x - 6) \div (x - 2)$

There are three methods to find the remainder: long division, synthetic division and remainder theorem. Since we were not asked to divide or find the quotient, the remainder theorem is the quickest way to find the remainder. Let $f(x) = x^3 - 3x^2 + 8x - 6$, the remainder is given by $f(2) = 6$.

Problem 7

Is $x - 3$ a factor of $2x^3 - 12x - 17$? How do you know?

Using the factor theorem, we know an expression is a factor if the remainder is 0. Let $f(x) = 2x^3 - 12x - 17$. Then $x - 3$ is a factor if and only if $f(3) = 0$. However, $f(3) = 1$, so $x - 3$ is not a factor.

Problem 8

Find the possible rational zeros of $f(x) = x^{19} + 14x - 4$

The possible zeros are $\frac{p}{q} = \frac{-4}{1} = \frac{\text{factors of } -4}{\text{factors of } 1} = \frac{\pm1, \pm2, \pm4}{\pm1}$
$= \pm1, \pm2, \pm4$

Problem 9

Find the possible rational zeros of $f(x) = 4x^{27} + 12x^{10} - x + 6$.

The possible zeros are $\frac{p}{q} = \frac{6}{4} = \frac{\text{factors of } 6}{\text{factors of } 4} = \frac{\pm1, \pm2, \pm3, \pm6}{\pm1, \pm2, \pm4}$
$= \pm1, \pm\frac{1}{2}, \pm\frac{1}{4}, \pm2, \pm3, \pm\frac{3}{2}, \pm\frac{3}{4}, \pm6$

Problem 10

Factor $f(x) = x^4 - x^3 - 7x^2 + x + 6$

To factor, you can use long or synthetic division. Since synthetic division is faster, I will demonstrate this method. First, find the possible zeros.

Then use this list to guess and check synthetic division to help you factor. Remember, we know a value is a factor if we get remainder zero.

The possible zeros are $\frac{p}{q} = \frac{6}{1} = \frac{\text{factors of 6}}{\text{factors of 1}} = \frac{\pm 1, \pm 2, \pm 3, \pm 6}{\pm 1} = \pm 1, \pm 2, \pm 3, \pm 6$

$$\begin{array}{r|rrrrr} & 1 & -1 & -7 & 1 & 6 \\ 1 & & 1 & 0 & -7 & -6 \\ \hline & 1 & 0 & -7 & -6 & 0 \end{array}$$

We got remainder 0 on the first try! If 1 is the zero, then $(x-1)$ is the factor. So, $f(x) = (x-1)(x^3 - 7x - 6)$. Repeat the process, to factor $x^3 - 7x - 6$ next.

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & -6 \\ 1 & & 1 & 1 & -6 \\ \hline & 1 & 1 & -6 & -12 \end{array}$$

We did not get remainder 0, so guess again.

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & -6 \\ 2 & & 2 & 4 & -6 \\ \hline & 1 & 2 & -3 & -12 \end{array}$$

We did not get remainder 0, so guess again.

$$\begin{array}{r|rrrr} & 1 & 0 & -7 & -6 \\ 3 & & 3 & 9 & 6 \\ \hline & 1 & 3 & 2 & 0 \end{array}$$

We got remainder 0! (Three guesses, not bad!). If 3 is the zero, then $(x-3)$ is the factor. Putting it all together we have $f(x) = (x-1)(x-3)(x^2+3x+2)$. Now that I only have a simple trinomial left, I can use our normal factoring techiniques from the beginning of the semester, $f(x) = (x-1)(x-3)(x+2)(x+1)$, alternatively you can use synthetic division until it is factored completely.

Final answer: $f(x) = (x-1)(x-3)(x+2)(x+1)$