Activity 2.4 - Answer Key

Problem 1

True or False: A rational function always has a horizontal asymptote.

False. For example, a rational function where the degree of the numerator is greater than the degree of the denominator does not have a HA.

Problem 2

Find the domain: $f(x) = \dfrac{3x-2}{x^2-3x+2}$

The domain of a rational function is all real numbers except where the function is undefined. A fraction is defined everywhere except when the denominator is 0. The denominator factors as $(x-1)(x-2)$, and thus has zeros at $x = 1, 2$. So, the domain is all real numbers except 1 and 2. In interval notation, we write properly: $(-\infty, 1) \cup (1, 2) \cup (2, \infty)$

Problem 3

Find the VA(s): $f(x) = \dfrac{x^2-6x+8}{x^2-x-12}$

The vertical asymptote(s) occur where the simplified functions denominator is set equal to 0.

$$f(x) = \frac{(x-4)(x-2)}{(x-4)(x+3)} = \frac{x-2}{x+3}, x \neq 4$$

So, the VA is at $x = -3$

At $x = 4$ we have a hole.

Problem 4

What are the differences between finding the domain and VA?

Both require finding the real zeros of the denominator, but finding the VAs requires an extra step of making sure the real zero cannot be canceled out by a zero in the numerator (of the same or greater multiplicity). In more simple language, the domain is the **original** function's denominator set equal to 0 (and write your answer in interval notation) and the VA is the **simplified** function's denominator set equal to 0 (and write your answer in the form of a vertical line "$x = \#$").

Problem 5

Find the HA: $f(x) = \dfrac{x+1}{x^3-5}$

The degree on top is smaller, so the HA is $y = 0$.

Problem 6

Find the HA: $f(x) = \dfrac{2x+1}{3x-5}$

The numerator and denominator have same degree, so the HA is the quotient of the leading coefficients, $y = \frac{2}{3}$.

Problem 7

Find the HA: $f(x) = \dfrac{2x^2+1}{x-5}$

The degree on top is bigger, so there is no HA.

Problem 8

True or False: The following rational expression has a slant asymptote:
$f(x) = \dfrac{3x^4+x}{x^2+3x-5}$

False, a slant asymptote exists only when the degree of the numerator is exactly 1 greater than that of the denominator.

Problem 9

Find the SA: $f(x) = \dfrac{x^3 - 1}{x^2}$

$$
\begin{array}{r|l}
 & x \\
\hline
x^2 & x^3 - 1 \\
 & -x^3 \\
\hline
 & -1
\end{array}
$$

Long division yields $f(x) = x + \frac{-1}{x^2}$, so the slant asymptote is just the quotient of the long division, so the SA is $y = x$.

Problem 10

Graph: $f(x) = \dfrac{-2x + 2}{x - 2}$

Hint: Find the domain, VA, HA, x and y intercepts, number line test, and then graph.

Domain: $(-\infty, 2) \cup (2, \infty)$

VA: $x = 2$

HA: $y = -2$

x-intercept(s): $(1, 0)$

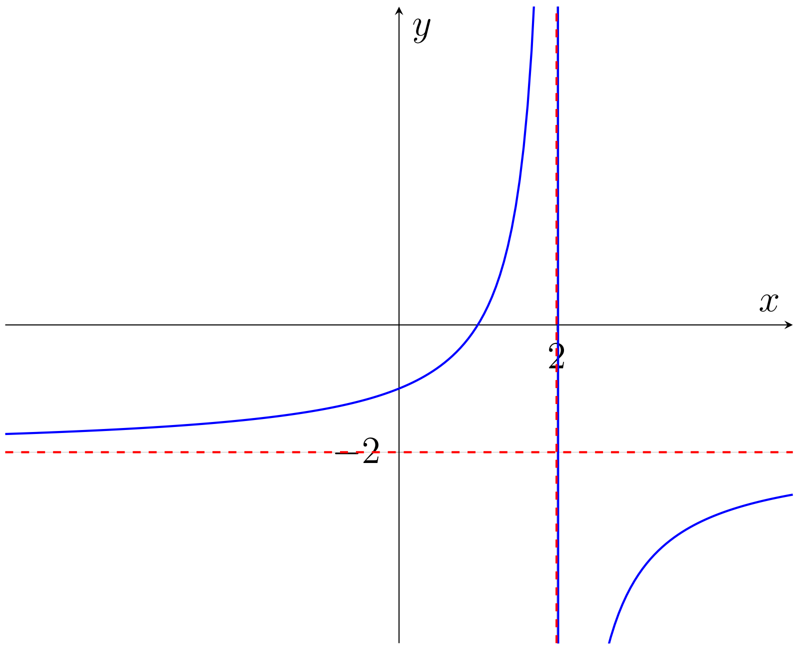
y-intercept: $(0, -1)$

Long Division:

$$
\begin{array}{r|l}
 & -2 \\
\hline
x - 2 & -2x + 2 \\
 & 2x - 4 \\
\hline
 & -2
\end{array}
$$

Long Division yields: $f(x) = -2 + \frac{-2}{x-2}$. Complete a number line test around the zeros of the numerator and denominator of the remainder function. In this case, the remainder function is $\frac{-2}{x-2}$. On the number line, test around 2.

NLT: + -



Problem 11

Graph: $f(x) = \dfrac{x^2}{(x-2)(x+3)}$

Hint: Find the domain, VA, HA, x and y intercepts, number line test, and then graph.

Domain: $(-\infty, -3) \cup (-3, 2) \cup (2, \infty)$

VA: $x = 2$ and $x = 3$

HA: $y = 1$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

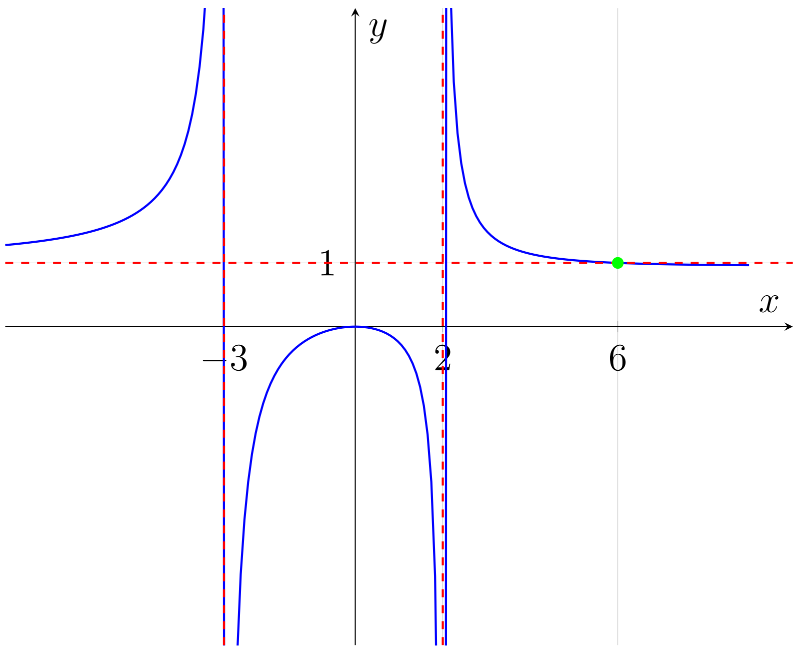
Long Division:

$$\begin{array}{r} 1 \\ x^2+x-6\overline{)\ x^2 \qquad\qquad} \\ -x^2-x+6 \\ \hline -x+6 \end{array}$$

Long Division yields: $f(x) = 1 + \frac{-x+6}{(x-2)(x+3)}$. Complete a number line test around the zeros of the numerator and denominator of the remainder function. In this case, the remainder function is $\frac{-x+6}{(x-2)(x+3)}$. On the number line, test around 2, 3 and 6.

NLT: + - + -

At $x = 6$, the graph crosses the HA.



Problem 12

The NLT when graphing a rational function tests whether the function is above or below what?

The NLT tests whether the function is above or below the HA (or SA).