Activity 3.4 - Answer Key

Problem 1

Solve: $3^{2x+1} = 27$

Step 1: Express 27 as a power of 3:

$$27 = 3^3$$

Step 2: Substitute 3^3 for 27 in the equation:

$$3^{2x+1} = 3^3$$

Step 3: Since the bases are the same, set the exponents equal:

$$2x + 1 = 3$$

Step 4: Solve for x:

$$2x + 1 = 3$$

$$2x = 2$$

$$x = 1$$

So, the solution is:

$$\boxed{x = 1}$$

Problem 2

Solve: $5^{2-x} = 3^x$

In this problem, we can't make the bases the same. To remove an exponent, we need a log (or ln).

$$\log(5^{2-x}) = \log(3^x)$$

$$(2 - x)\log(5) = x\log(3)$$

$$2\log(5) - x\log(5) = x\log(3)$$

$$2\log(5) = x\log(3) + x\log(5)$$

$$2\log(5) = x[\log(3) + \log(5)]$$
$$x = \frac{2\log(5)}{\log(3) + \log(5)}$$

Problem 3

Solve: $\frac{1}{3}\log_3(2x-1) + \frac{2}{3} = 1$

Step 1: Isolate the logarithmic term

Subtract $\frac{2}{3}$ from both sides:

$$\frac{1}{3}\log_3(2x-1) = 1 - \frac{2}{3}$$

Simplify the right side:

$$\frac{1}{3}\log_3(2x-1) = \frac{1}{3}$$

Step 2: Eliminate the fraction

Multiply both sides by 3 to get rid of the fraction:

$$\log_3(2x-1) = 1$$

Step 3: Rewrite the logarithmic equation in exponential form

Recall that $\log_b a = c$ is equivalent to $b^c = a$. Therefore, we can rewrite the equation:

$$2x - 1 = 3^1$$

Simplify the right side:

$$2x - 1 = 3$$

Step 4: Solve for x

Add 1 to both sides:

$$2x = 4$$

Divide both sides by 2:

$$x = 2$$

Conclusion

The solution to the equation is:

$$\boxed{x = 2}$$

Remember to plug 2 back in and verify it is in the domain. It works for this problem.

Problem 4

Solve: $5^{2x} - 4 \cdot 5^x = 21$

Step 1: Substitute $y = 5^x$

Let $y = 5^x$. Then $5^{2x} = (5^x)^2 = y^2$. The equation becomes:

$$y^2 - 4y = 21$$

Step 2: Rewrite as a Quadratic Equation

Rewrite the equation as a general quadratic equation:

$$y^2 - 4y - 21 = 0$$

Step 3: Solve the Quadratic Equation

Solve the quadratic equation using the quadratic formula $y = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$, where $a = 1$, $b = -4$, and $c = -21$ (or you can solve via factoring):

$$y = \frac{-(-4) \pm \sqrt{(-4)^2 - 4 \cdot 1 \cdot (-21)}}{2 \cdot 1}$$
$$y = \frac{4 \pm \sqrt{16 + 84}}{2}$$
$$y = \frac{4 \pm \sqrt{100}}{2}$$
$$y = \frac{4 \pm 10}{2}$$

This gives us two solutions:

$$y = \frac{4 + 10}{2} = 7$$
$$y = \frac{4 - 10}{2} = -3$$

Since $y = 5^x$ and 5^x is always positive, we discard $y = -3$ as it is not valid.

So, we have:

$$y = 7$$

Step 4: Back-Substitute $y = 5^x$

Back-substitute $y = 7$ to find x:

$$5^x = 7$$

Step 5: Solve for x

Take the logarithm of both sides:

$$\log(5^x) = \log(7)$$

Using the property of logarithms $\log(a^b) = b\log(a)$:

$$x\log(5) = \log(7)$$

Solve for x:

$$x = \frac{\log(7)}{\log(5)}$$

Conclusion

The solution to the equation is:

$$\boxed{x = \frac{\log(7)}{\log(5)}}$$

Problem 5

Solve: $e^{2x} - 5e^x + 4 = 0$

Step 1: Substitute $y = e^x$

Let $y = e^x$. Then $e^{2x} = (e^x)^2 = y^2$. The equation becomes:

$$y^2 - 5y + 4 = 0$$

Step 2: Solve the Quadratic Equation

Solve the quadratic equation using the quadratic formula $y = \frac{-b\pm\sqrt{b^2-4ac}}{2a}$, where $a = 1$, $b = -5$, and $c = 4$ (or you can solve via factoring):

$$y = \frac{-(-5) \pm \sqrt{(-5)^2 - 4\cdot 1\cdot 4}}{2\cdot 1}$$
$$y = \frac{5 \pm \sqrt{25-16}}{2}$$
$$y = \frac{5 \pm \sqrt{9}}{2}$$

$$y = \frac{5 \pm 3}{2}$$

This gives us two solutions:

$$y = \frac{5 + 3}{2} = 4$$

$$y = \frac{5 - 3}{2} = 1$$

Step 3: Back-Substitute $y = e^x$

Back-substitute $y = 4$ and $y = 1$ to find x:

For $y = 4$:

$$e^x = 4$$

Take the natural logarithm of both sides:

$$x = \ln(4)$$

For $y = 1$:

$$e^x = 1$$

Take the natural logarithm of both sides:

$$x = \ln(1) = 0$$

Conclusion

The solutions to the equation are:

$$\boxed{x = \ln(4) \text{ or } x = 0}$$

Problem 6

Solve: $\log(x+8) + \log(x-1) = 1$

Step 1: Combine the Logarithms

Use the property of logarithms $\log(a) + \log(b) = \log(ab)$ to combine the logarithms on the left side:

$$\log((x+8)(x-1)) = 1$$

Step 2: Rewrite in Exponential Form

Rewrite the logarithmic equation in exponential form. Recall that $\log_b(a) = c$ is equivalent to $b^c = a$. Here, the base is 10:

$$10^1 = (x+8)(x-1)$$

Step 3: Simplify the Equation

Simplify the right side:

$$10 = (x+8)(x-1)$$

$$10 = x^2 + 8x - x - 8$$

$$10 = x^2 + 7x - 8$$

Step 4: Solve the Quadratic Equation

Rearrange the equation to the form $ax^2 + bx + c = 0$:

$$x^2 + 7x - 18 = 0$$

Factor the quadratic equation:

$$(x+9)(x-2)=0$$

Set each factor equal to zero and solve for x:

$$x+9=0 \quad \text{or} \quad x-2=0$$

$$x=-9 \quad \text{or} \quad x=2$$

Step 5: Check for Extraneous Solutions

Since logarithms are undefined for non-positive arguments, we need to check the potential solutions in the original logarithmic expressions.

For $x=-9$:

$$\log(-9+8)+\log(-9-1)=\log(-1)+\log(-10) \quad \text{(undefined)}$$

For $x=2$:

$$\log(2+8)+\log(2-1)=\log(10)+\log(1)$$

$$\log(10)+\log(1)=1+0=1$$

This satisfies the original equation.

So, $x=-9$ is an extraneous solution and should be discarded.

Conclusion

The solution to the equation is:

$$\boxed{x=2}$$

Problem 7

Solve: $\log_2(x+10) - \log_2(x-5) = 4$

Step 1: Combine the Logarithms

Use the property of logarithms $\log_b(a) - \log_b(b) = \log_b\left(\frac{a}{b}\right)$ to combine the logarithms on the left side:

$$\log_2\left(\frac{x+10}{x-5}\right) = 4$$

Step 2: Rewrite in Exponential Form

Rewrite the logarithmic equation in exponential form. Recall that $\log_b(a) = c$ is equivalent to $b^c = a$. Here, the base is 2:

$$2^4 = \frac{x+10}{x-5}$$

Step 3: Simplify the Equation

Simplify the right side:

$$16 = \frac{x+10}{x-5}$$

Multiply both sides by $x-5$ to clear the fraction:

$$16(x-5) = x+10$$

Distribute 16 on the left side:

$$16x - 80 = x + 10$$

Step 4: Solve for x

Move all terms involving x to one side and constant terms to the other side:

$$16x - x = 10 + 80$$

$$15x = 90$$

Divide both sides by 15:

$$x = 6$$

Step 5: Check the Solution

Ensure that the solution satisfies the conditions $x+10 > 0$ and $x-5 > 0$:

$$x + 10 = 6 + 10 = 16 > 0$$

$$x - 5 = 6 - 5 = 1 > 0$$

Both conditions are satisfied.

Conclusion

The solution to the equation is:

$$\boxed{x = 6}$$

Problem 8

Solve: $2^{3x+5} = 7^{x-4}$

Step 1: Take the Natural Logarithm of Both Sides

To solve for x, take the natural logarithm of both sides:

$$\ln(2^{3x+5}) = \ln(7^{x-4})$$

Step 2: Use the Power Rule for Logarithms

Use the power rule for logarithms $\ln(a^b) = b\ln(a)$ to bring the exponents down:

$$(3x + 5)\ln(2) = (x - 4)\ln(7)$$

Step 3: Distribute the Logarithms

Distribute $\ln(2)$ and $\ln(7)$ to both terms:

$$3x\ln(2) + 5\ln(2) = x\ln(7) - 4\ln(7)$$

Step 4: Collect Terms Involving x

Move all terms involving x to one side and constant terms to the other side:

$$3x\ln(2) - x\ln(7) = -4\ln(7) - 5\ln(2)$$

Factor out x from the left side:

$$x(3\ln(2) - \ln(7)) = -4\ln(7) - 5\ln(2)$$

Step 5: Solve for x

Divide both sides by $(3\ln(2) - \ln(7))$ to isolate x:

$$x = \frac{-4\ln(7) - 5\ln(2)}{3\ln(2) - \ln(7)}$$

Conclusion

The solution to the equation is:

$$\boxed{x = \frac{-4\ln(7) - 5\ln(2)}{3\ln(2) - \ln(7)}}$$

Problem 9

Solve: $\log x + \log(x - 99) = 2$

When two logs of the same base are added, the argument is multiplied, thus

$$\log x + \log(x - 99) = \log[x(x - 99)] = 2$$

$$\implies \log\left[x^2 - 99x\right] = 2$$

We exponentiate in order to remove the log

$$\implies x^2 - 99x = 10^2$$

$$\implies x^2 - 99x - 100 = 0$$

$$(x - 100)(x + 1) = 0$$

$$x = 100 \quad x = -1$$

Plug in your solutions and check to make sure they are in the domain of the logarithm. Recall, you can only take the log of a positive number. You will find 100 works, but -1 does not. -1 is extraneous. The final solution is $\boxed{x = 100}$.