Activity 4.3 - Answer Key

Problem 1

Fill in the blanks for each part:

A) In quadrant I: ALL functions are positive.

B) In quadrant II: SIN and CSC are positive.

C) In quadrant III: TAN and COT are positive.

D) In quadrant IV: COS and SEC are positive.

Problem 2

In which two quadrants is $\tan(\theta)$ negative?

$\tan(\theta)$ is negative in the second and fourth quadrants.

Problem 3

If $\sin\theta < 0$ and $\sec\theta < 0$, which quadrant is θ in?

If $\sin\theta$ is negative, θ must be in either the third or fourth quadrant. If $\sec\theta$ is also negative, it means θ is either in the second or third quadrant. Therefore, when $\sin\theta < 0$ and $\sec\theta < 0$, θ is in the third quadrant.

Problem 4

If $\sin\theta > 0$ and $\tan\theta > 0$, which quadrant is θ in?

If $\sin\theta$ is positive, it must be in either the first or second quadrant. If $\tan\theta$ is also positive, it means θ is either in the first or fourth quadrant. Therefore, when $\sin\theta > 0$ and $\tan\theta > 0$, θ is in the first quadrant.

Problem 5

Given the point $(3, -4)$, find the six trigonometric functions.

This point is not on the unit circle, so you must find the radius first.

$(3)^2 + (-4)^2 = r^2$

$9 + 16 = r^2$

$25 = r^2$

$r = 5$

Now the six trig functions are:

$\sin(\theta) = \frac{y}{r} = \frac{-4}{5}$

$\cos(\theta) = \frac{x}{r} = \frac{3}{5}$

$\tan(\theta) = \frac{y}{x} = \frac{-4}{3}$

$\csc(\theta) = \frac{r}{y} = \frac{5}{-4}$

$\sec(\theta) = \frac{r}{x} = \frac{5}{3}$

$\cot(\theta) = \frac{x}{y} = \frac{3}{-4}$

Problem 6

If $\cos\theta = -\frac{7}{25}$ and θ is in the second quadrant, find $\tan\theta$.

Use the trigonometric identity:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

Because $\cos\theta = -\frac{7}{25}$ and θ is in the second quadrant where $\cos\theta$ is negative, we need to find the positive value of $\sin\theta$ in the second quadrant. Using the Pythagorean identity for sine and cosine:

$$\sin^2\theta + \cos^2\theta = 1$$

Find $\sin\theta$ using this equation:

$$\sin^2\theta = 1 - \cos^2\theta$$

$$\sin^2\theta = 1 - \left(-\frac{7}{25}\right)^2$$
$$\sin^2\theta = 1 - \frac{49}{625}$$
$$\sin^2\theta = \frac{625}{625} - \frac{49}{625}$$
$$\sin^2\theta = \frac{576}{625}$$

Next, because θ is in the second quadrant, $\sin\theta$ has to be positive, so you need to take the positive square root:

$$\sin\theta = \sqrt{\frac{576}{625}}$$
$$\sin\theta = \frac{24}{25}$$

Next, find $\tan\theta$ by dividing $\sin\theta$ by $\cos\theta$:

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$
$$\tan\theta = \frac{\frac{24}{25}}{-\frac{7}{25}}$$
$$\tan\theta = -\frac{24}{7}$$

Problem 7

If $\cos\theta = \frac{3}{5}$ and $\tan\theta < 0$, find $\sin\theta$.

Use the fact that $\tan\theta = \frac{\sin\theta}{\cos\theta}$. Because $\tan\theta < 0$, it means that $\sin\theta$ and $\cos\theta$ will have to have opposite signs. Since $\cos\theta = \frac{3}{5}$, and it's positive, if $\tan\theta$ is negative, $\sin\theta$ has to be negative as well. This means that $\sin\theta$ is in the fourth quadrant.

To solve for sine, use the Pythagorean identity for sine and cosine:

$$\sin^2\theta + \cos^2\theta = 1$$

$$\sin^2\theta = 1 - \cos^2\theta$$
$$\sin^2\theta = 1 - \left(\frac{3}{5}\right)^2$$
$$\sin^2\theta = 1 - \frac{9}{25}$$
$$\sin^2\theta = \frac{25}{25} - \frac{9}{25}$$
$$\sin^2\theta = \frac{16}{25}$$

Take the negative square root because $\sin\theta$ is negative in the fourth quadrant:

$$\sin\theta = -\sqrt{\frac{16}{25}}$$
$$\sin\theta = -\frac{4}{5}$$

Problem 8

Determine if $\sec\theta$ is an even or odd function.

The secant function, $\sec\theta$ is even. An even function is defined as:

$$f(-x) = f(x)$$

To prove this, $\sec\theta$ can be re-written as:

$$\sec(-\theta) = \frac{1}{\cos(-\theta)}$$

Using the even property of cosine ($\cos(-\theta) = \cos(\theta)$):

$$\sec(-\theta) = \frac{1}{\cos(\theta)} = \sec(\theta)$$

Activity 4.3 - Answer Key

Problem 9

Find the measure of the reference angle for a 120° angle.

$180^\circ - 120^\circ = 60^\circ$

Problem 10

Find the measure of the reference angle for a 290° angle.

$360^\circ - 290^\circ = 70^\circ$

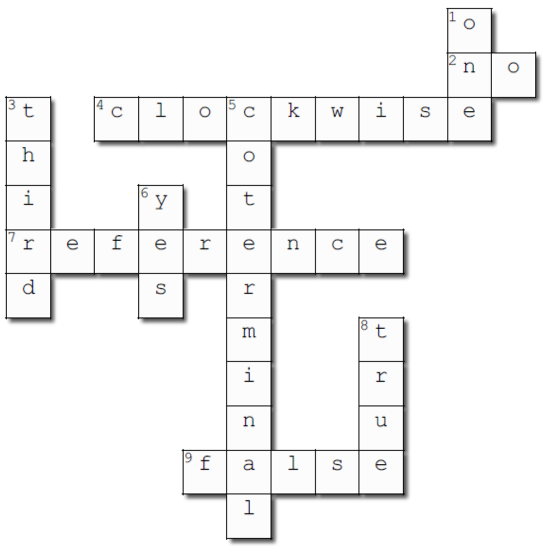
Problem 11

Find the measure of the reference angle for a 200° angle.

$200^\circ - 180^\circ = 20^\circ$

Crossword Puzzle

Did you complete the crossword puzzle?



Across

2. Yes or No: theta = r(s) (**no**)
4. A negative angle is formed by rotating the initial side in this direction. (**clockwise**)
7. The acute angle formed by the terminal side and the x-axis is the _____ angle. (**reference**)
9. True or False: sin(x) is an even function. (**false**)

Down

1. A unit circle is centered at the origin with radius value of this. (**one**)
3. Tangent is positive in the first and _____ quadrants. (**third**)
5. Two angles with the same initial and terminal sides are this. (**coterminal**)
6. Yes or No: Will the unit circle be provided on the exam? (**yes**)
8. True or False: The 6 trig functions for pi and -pi are the same. (**true**)