Activity 4.4 - Answer Key

Problem 1

What is the period of the sin and cos function?

The period for both functions is 2π.

Problem 2

Determine the amplitude of $y = -6\sin(x)$

Use the equation $y = a\sin\left[b(x - c) + d\right]$, where the amplitude is $|a| = a$.

The amplitude is $|-6| = 6$.

Problem 3

What is the phase shift of $y = \cos(2x - \pi)$?

First, put the function into the equation form of $y = a\sin\left[b(x - c) + d\right]$, where c is the phase shift.

$$y = \cos\left(2\left(x - \frac{\pi}{2}\right)\right)$$

Then, the phase shift is $\frac{\pi}{2}$ to the right.

Problem 4

What is the phase shift of $y = \sin\left(\frac{1}{6}x\right)$?

Phase shift is 0 since there is no c value in this function.

Problem 5

What is the vertical shift of $y = \sin(x - \frac{\pi}{2}) + 7$?

Use the equation $y = a\sin\left[b(x - c) + d\right]$, where d is the vertical shift.

The vertical shift is up 7.

Problem 6

Graph $y = \sin\left(\frac{1}{2}x\right)$

Step 1:

$$\text{Amplitude} = |1| = 1$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$\text{Phase shift} = 0$$

$$\text{Vertical shift} = 0$$

Step 2: Find the domain of 1 cycle using $[c, c + \frac{2\pi}{b}]$.

$$[0, 0 + 4\pi]$$

$$[0, 4\pi]$$

Step 3: Determine the key points,
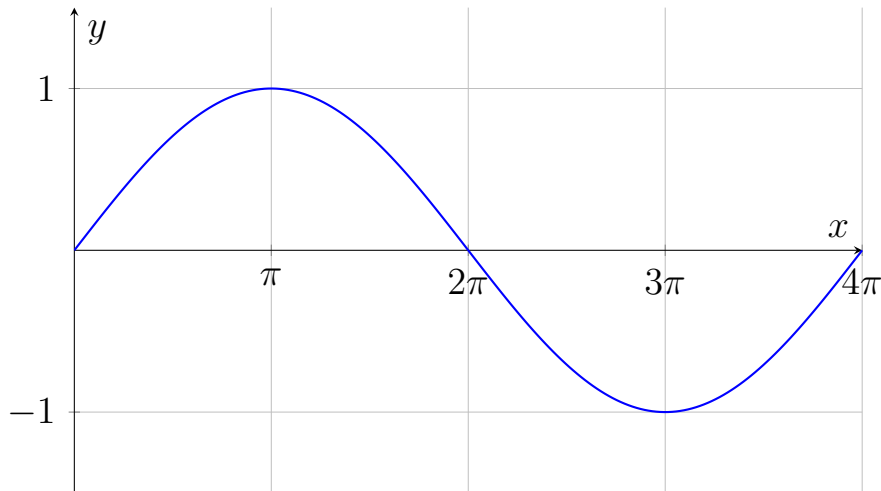
$$\frac{1}{4}(4\pi) = \pi$$

$$0, \quad 0 + \pi = \pi, \quad \pi + \pi = 2\pi, \quad 2\pi + \pi = 3\pi, \quad 3\pi + \pi = 4\pi$$

$$\text{Key points} : 0, \pi, 2\pi, 3\pi, 4\pi$$

Step 4: X/Y chart and graph $\rightarrow$

X	Y
0	0
π	1
2π	0
3π	-1
4π	0

Problem 7

Graph $y = -\cos(x)$

Step 1:

$$\text{Amplitude} = |-1| = 1$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

$$\text{Phase shift} = 0$$

$$\text{Vertical shift} = 0$$

Step 2: Find the domain of 1 cycle using $[c, c + \frac{2\pi}{b}]$.

$$[0, 0 + 2\pi]$$

$$[0, 2\pi]$$

Step 3: Determine the key points,

$$\frac{1}{4}(2\pi) = \frac{\pi}{2}$$

$$0 + \frac{\pi}{2} = \frac{\pi}{2},$$
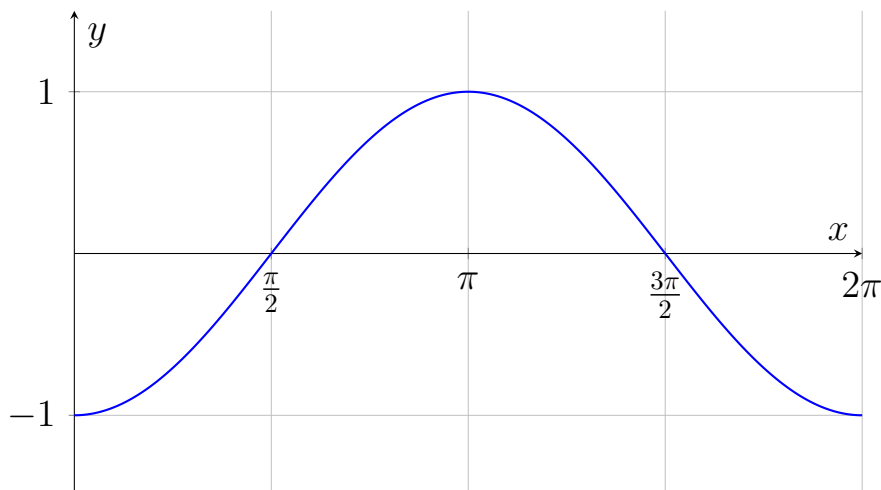$$\frac{\pi}{2} + \frac{\pi}{2} = \pi,$$
$$\pi + \frac{\pi}{2} = \frac{3\pi}{2},$$
$$\frac{3\pi}{2} + \frac{\pi}{2} = \frac{4\pi}{2} = 2\pi.$$

$$\text{Key points}: 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$$

Step 4: X/Y chart and graph $\rightarrow$

X	Y
0	-1
$\frac{\pi}{2}$	0
π	1
$\frac{3\pi}{2}$	0
2π	-1

Problem 8

Graph $f(x) = -5\sin\left(x - \frac{\pi}{3}\right)$

Step 1:

$$\text{Amplitude} = |-5| = 5$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$$

$$\text{Phase shift} = \frac{\pi}{3} \text{ (to the right)}$$

$$\text{Vertical shift} = 0$$

Step 2: Find the domain of 1 cycle using $[c, c + \frac{2\pi}{b}]$.

$$[\frac{\pi}{3}, \frac{\pi}{3} + 2\pi]$$

$$[\frac{\pi}{3}, \frac{7\pi}{3}]$$

Step 3: Determine the key points,

$$\frac{1}{4}(2\pi) = \frac{\pi}{2}$$

$$\frac{\pi}{3} + \frac{\pi}{2} = \frac{5\pi}{6},$$
$$\frac{5\pi}{6} + \frac{\pi}{2} = \frac{8\pi}{6},$$
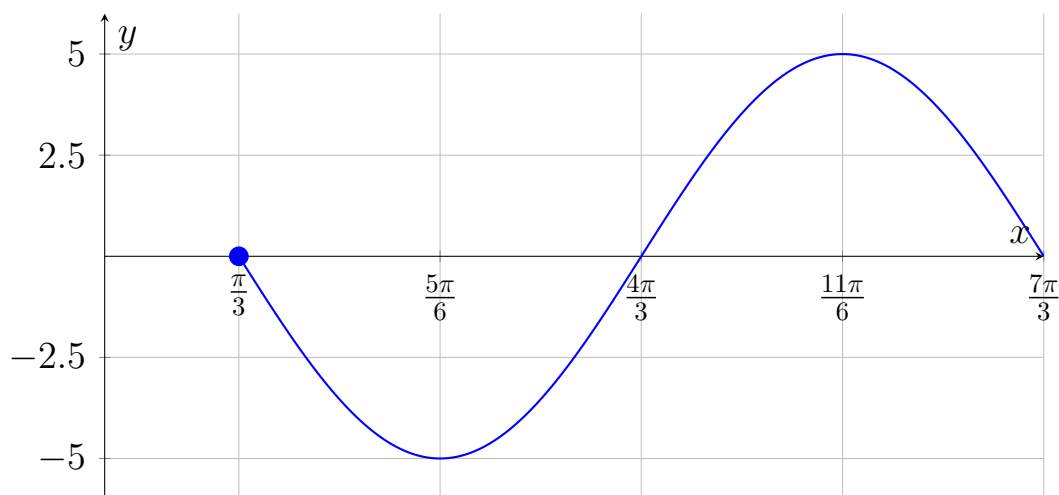$$\frac{8\pi}{6} + \frac{\pi}{2} = \frac{11\pi}{6},$$
$$\frac{11\pi}{6} + \frac{\pi}{2} = \frac{14\pi}{6} = \frac{7\pi}{3}.$$

$$\text{Key points}: \frac{\pi}{3}, \frac{5\pi}{6}, \frac{8\pi}{6}, \frac{11\pi}{6}, \frac{14\pi}{6}$$

Step 4: X/Y chart and graph →

X	Y
$\frac{\pi}{3}$	0
$\frac{5\pi}{6}$	-5
$\frac{4\pi}{3}$	0
$\frac{11\pi}{6}$	5
$\frac{7\pi}{3}$	0

Problem 9

Graph $y = 2\cos\left(3x + \frac{\pi}{2}\right) - 1$

Step 1:

First put the equation into the form of $y = a\sin\left[b(x - c) + d\right]$

$$2\cos\left(3\left(x + \frac{\pi}{6}\right)\right) - 1$$

$$\text{Amplitude} = |2| = 2$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{3}$$

$$\text{Phase shift} = -\frac{\pi}{6} \text{ (to the left)}$$

$$\text{Vertical shift} = -1 \text{ (down)}$$

Step 2: Find the domain of 1 cycle using $\left[c, c + \frac{2\pi}{b}\right]$.

$$\left[-\frac{\pi}{6}, -\frac{\pi}{6} + \frac{2\pi}{3}\right]$$

$$\left[-\frac{\pi}{6}, \frac{\pi}{2}\right]$$

Step 3: Determine the key points,

$$\frac{1}{4}\left(\frac{2\pi}{3}\right) = \frac{\pi}{6}$$

$$-\frac{\pi}{6} + \frac{\pi}{6} = 0,$$
$$0 + \frac{\pi}{6} = \frac{\pi}{6},$$
$$\frac{\pi}{6} + \frac{\pi}{6} = \frac{2\pi}{6},$$
$$\frac{2\pi}{6} + \frac{\pi}{6} = \frac{3\pi}{6} = \frac{\pi}{2}.$$

$$\text{Key points}: -\frac{\pi}{6}, 0, \frac{\pi}{6}, \frac{2\pi}{6}, \frac{3\pi}{6}$$

Step 4: X/Y chart and graph $\rightarrow$

X	Y
$\frac{-\pi}{6}$	1
0	-1
$\frac{\pi}{6}$	-3
$\frac{\pi}{3}$	-1
$\frac{\pi}{2}$	1