

Activity 4.5 - Answer Key

Problem 1

What is the period of the sec and csc functions?

The period for both functions is 2π .

Problem 2

What is the period of cot?

The period of cot is π .

Problem 3

What is the domain of cot?

The domain of cot is all real numbers except $\sin(x) = 0$, $x \neq n\pi$

Problem 4

Find the LVA and RVA of $y = \tan(x - \frac{\pi}{4})$

To find LVA \rightarrow

Use LVA equation: $b(x - c) = -\frac{\pi}{2}$

$$x - \frac{\pi}{4} = -\frac{\pi}{2}$$

$$x = -\frac{\pi}{2} + \frac{\pi}{4}$$

$$x = -\frac{2\pi}{4} + \frac{\pi}{4}$$

$$\text{LVA is } x = -\frac{\pi}{4}$$

To find RVA \rightarrow

Use RVA equation: $b(x - c) = \frac{\pi}{2}$

$$x - \frac{\pi}{4} = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$x = \frac{2\pi}{4} + \frac{\pi}{4}$$

$$\text{RVA is } x = \frac{3\pi}{4}$$

Activity 4.5 - Answer Key

Problem 5

Find the LVA and RVA of $y = \tan(2x - \frac{\pi}{4}) - 3$

To find LVA \rightarrow

Use LVA equation: inside stuff = $-\frac{\pi}{2}$

$$2x - \frac{\pi}{4} = -\frac{\pi}{2}$$

$$2x = -\frac{\pi}{2} + \frac{\pi}{4}$$

$$2x = -\frac{\pi}{4}$$

$$\text{LVA is } x = -\frac{\pi}{8}$$

To find RVA \rightarrow

Use RVA equation: inside stuff = $\frac{\pi}{2}$

$$2x - \frac{\pi}{4} = \frac{\pi}{2}$$

$$2x = \frac{\pi}{2} + \frac{\pi}{4}$$

$$2x = \frac{3\pi}{4}$$

$$\text{RVA is } x = \frac{3\pi}{8}$$

Problem 6

Graph $y = 3 \tan(x - \frac{\pi}{3}) + 2$

Step 1:

$$\text{Vertical stretch} = |a| = |3| = 3$$

$$\text{Period} = \frac{\pi}{b} = \pi$$

Activity 4.5 - Answer Key

$$\text{Phase shift} = \frac{\pi}{3} \text{ (to the right)}$$

$$\text{Vertical shift} = 2 \text{ (up)}$$

Step 2:

To find LVA \rightarrow Use LVA equation: inside stuff = $-\frac{\pi}{2}$

$$x - \frac{\pi}{3} = -\frac{\pi}{2}$$

$$x = -\frac{\pi}{2} + \frac{\pi}{3}$$

$$x = -\frac{3\pi}{6} + \frac{2\pi}{6}$$

$$x = -\frac{\pi}{6}$$

To find RVA \rightarrow Use RVA equation: inside stuff = $\frac{\pi}{2}$

$$x - \frac{\pi}{3} = \frac{\pi}{2}$$

$$x = \frac{\pi}{2} + \frac{\pi}{3}$$

$$x = \frac{3\pi}{6} + \frac{2\pi}{6}$$

$$x = \frac{5\pi}{6}$$

Step 3: Find key points \rightarrow First, find $\frac{1}{4}$ (period) = $\frac{1}{4}\pi = \frac{\pi}{4} = \#$.
Then, start with the LVA and keep adding, $\#$, until you get to the RVA
[remember your domain of 1 cycle is: $(-\frac{\pi}{6}, \frac{5\pi}{6}]$] to determine key points.

$$-\frac{\pi}{6} + \frac{\pi}{4} = \frac{\pi}{12} + \frac{\pi}{4} = \frac{4\pi}{12} + \frac{\pi}{4} = \frac{7\pi}{12} + \frac{\pi}{4} = \frac{10\pi}{12}$$

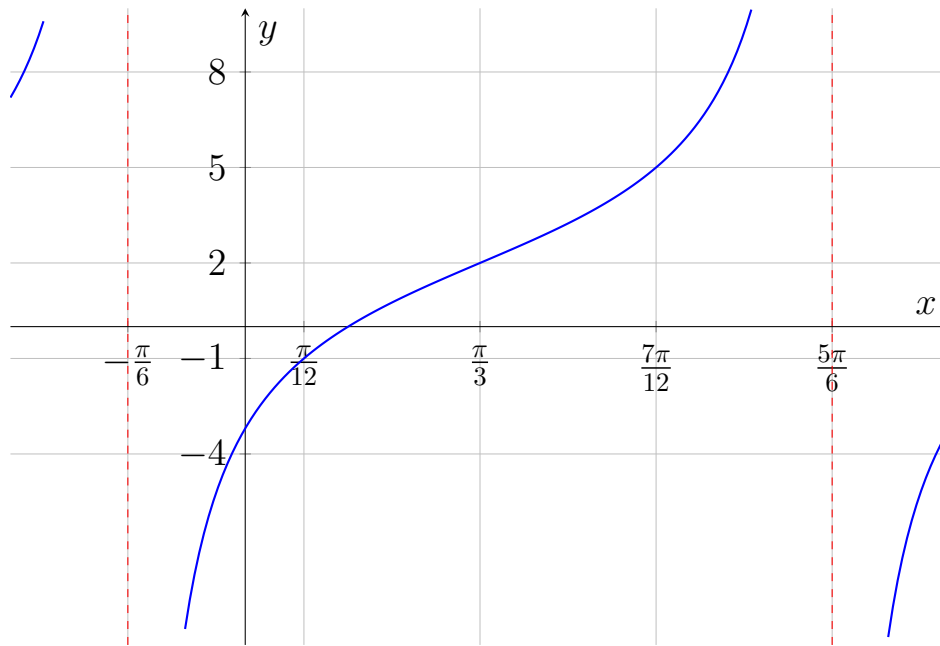
Key points:

$$-\frac{\pi}{6}, \quad \frac{\pi}{12}, \quad \frac{4\pi}{12}, \quad \frac{7\pi}{12}, \quad \frac{10\pi}{12}$$

Activity 4.5 - Answer Key

Step 4: X/Y chart and graph \rightarrow

X	Y
$-\frac{\pi}{6}$	LVA
$\frac{\pi}{12}$	-1
$\frac{\pi}{3}$	2
$\frac{7\pi}{12}$	5
$\frac{5\pi}{6}$	RVA



Problem 7

Graph $y = \tan(2x + \pi)$

Step 1:

First put the function into the form $y = a \tan(b(x - c)) + d \rightarrow$

$$y = \tan\left(2\left(x + \frac{\pi}{2}\right)\right)$$

$$\text{Vertical stretch} = |1| = 1$$

$$\text{Period} = \frac{\pi}{b} = \frac{\pi}{2}$$

Activity 4.5 - Answer Key

$$\text{Phase shift} = -\frac{\pi}{2} \text{ (to the left)}$$

$$\text{Vertical shift} = 0$$

Step 2:

To find LVA \rightarrow Use LVA equation: $b(x - c) = -\frac{\pi}{2}$

$$2\left(x + \frac{\pi}{2}\right) = -\frac{\pi}{2}$$

$$x + \frac{\pi}{2} = -\frac{\pi}{4}$$

$$x = -\frac{\pi}{4} - \frac{2\pi}{4}$$

$$x = -\frac{3\pi}{4}$$

To find RVA \rightarrow Use RVA equation: $b(x - c) = \frac{\pi}{2}$

$$2\left(x + \frac{\pi}{2}\right) = \frac{\pi}{2}$$

$$x + \frac{\pi}{2} = \frac{\pi}{4}$$

$$x = \frac{\pi}{4} - \frac{2\pi}{4}$$

$$x = -\frac{\pi}{4}$$

Step 3: Find key points \rightarrow First, find $\frac{1}{4}$ (period) $= \frac{1}{4}\left(\frac{\pi}{2}\right) = \frac{\pi}{8} = \#$.
Then, start with the LVA and keep adding, $\#$, until you get to the RVA
[remember your domain of 1 cycle is: $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right)$] to determine key points.

$$-\frac{3\pi}{4} + \frac{\pi}{8} = -\frac{5\pi}{8} + \frac{\pi}{8} = -\frac{4\pi}{8} + \frac{\pi}{8} = -\frac{3\pi}{8} + \frac{\pi}{8} = -\frac{2\pi}{8} = -\frac{\pi}{4}$$

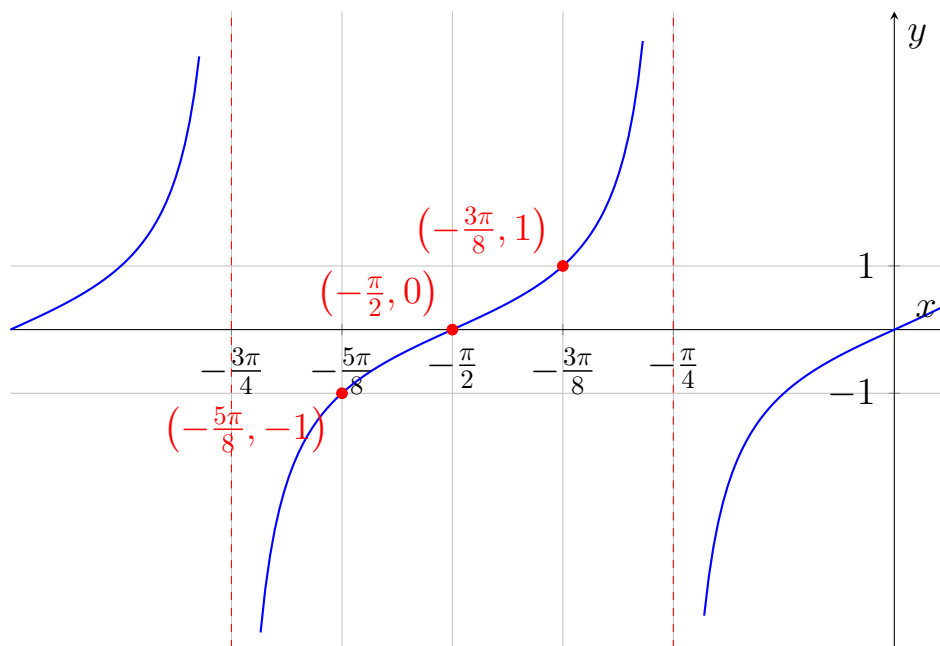
Key Points:

$$-\frac{3\pi}{4}, -\frac{5\pi}{8}, -\frac{4\pi}{8}, -\frac{3\pi}{8}, -\frac{2\pi}{8}$$

Activity 4.5 - Answer Key

Step 4: X/Y chart and graph \rightarrow

X	Y
$-\frac{3\pi}{4}$	LVA
$-\frac{5\pi}{8}$	-1
$-\frac{\pi}{2}$	0
$-\frac{3\pi}{8}$	1
$-\frac{\pi}{4}$	RVA



Problem 8

True or False: The non-vertical line makes an angle θ with the positive x -axis, then the slope of the line is given by $m = \tan \theta$.

True!

Problem 9

True or False: The zeros of $\tan(x)$ are all integer multiples of π .

True!

Activity 4.5 - Answer Key

Problem 10

True or False: The domain of $\tan(x)$ is all real numbers except multiples of π

False! The domain of $\tan(x)$ is all real numbers except odd multiples of $\frac{\pi}{2}$

Problem 11

What is the range of the $y = \csc(x)$ and $y = \sec(x)$ function?

$(-\infty, -1] \cup [1, \infty)$

Problem 12

What are the x-intercepts of $y = \csc(x)$ and $y = \sec(x)$ function?

There are no x-intercepts!