Activity 5.1 - Answer Key

Problem 1

State the three Pythagorean identities.

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \sec^2(\theta)$$

$$1 + \cot^2(\theta) = \csc^2(\theta)$$

Problem 2

Simplify: $(\sec(x) + \tan(x))(\sec(x) - \tan(x))$

$$(\sec(x) + \tan(x))(\sec(x) - \tan(x))$$

$$= \sec^2(x) - \sec(x)\tan(x) + \sec(x)\tan(x) - \tan^2(x)$$

$$= 1 + \tan^2(x) - \tan^2(x)$$

$$= 1$$

Problem 3

Simplify: $(1 + \tan(x))(1 - \tan(x)) + \sec^2(x)$

$$(1 + \tan(x))(1 - \tan(x)) + \sec^2(x)$$

$$= 1 - \tan^2(x) + \sec^2(x)$$

$$= 1 - \tan^2(x) + 1 + \tan^2(x)$$

$$= 2$$

Problem 4

Factor and Simplify: $\cos^4(x) - \sin^4(x)$

$$\cos^4(x) - \sin^4(x)$$

$$= (\cos^2(x) + \sin^2(x))(\cos^2(x) - \sin^2(x))$$

$$= (1)(\cos^2(x) - \sin^2(x))$$

$$= 1 - \sin^2(x) - \sin^2(x)$$

$$= 1 - 2\sin^2(x)$$

Problem 5

Verify: $\sin(x)\tan(x) + \cos(x) = \sec(x)$

$$\sin(x)\tan(x) + \cos(x) = \sec(x)$$

$$\sin(x)(\frac{\sin(x)}{\cos(x)}) + \cos(x) = \sec(x)$$

$$\frac{\sin^2(x)}{\cos(x)} + \cos(x) = \sec(x)$$

$$\frac{1 - \cos^2(x)}{\cos(x)} + \cos(x) = \sec(x)$$

$$\frac{1}{\cos(x)} - \frac{\cos^2(x)}{\cos(x)} + \cos(x) = \sec(x)$$

$$\sec(x) - \cos(x) + \cos(x) = \sec(x)$$

$$\sec(x) = \sec(x) \quad \checkmark$$

Activity 5.1 - Answer Key

Problem 6

Verify: $(\cos(x) - \sin(x))(\cos(x) + \sin(x)) = 1 - 2\sin^2(x)$

$$(\cos(x) - \sin(x))(\cos(x) + \sin(x)) = 1 - 2\sin^2(x)$$

$$\cos^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$1 - \sin^2(x) - \sin^2(x) = 1 - 2\sin^2(x)$$

$$1 - 2\sin^2(x) = 1 - 2\sin^2(x) \quad \checkmark$$

Problem 7

Verify: $\frac{1-4\cos^2(x)}{1-2\cos(x)} = 1 + 2cos(x)$

$$\frac{(1 + 2\cos(x))(1 - 2\cos(x))}{1 - 2\cos(x)} = 1 + 2\cos(x)$$

$$1 + 2\cos(x) = 1 + 2\cos(x) \quad \checkmark$$