

Activity 5.3 - Answer Key

Problem 1

Find the exact value: $2 \cos^2(165^\circ) - 1$

$$\begin{aligned} 2 \cos^2(165^\circ) - 1 &= \cos(2(165^\circ)) \\ &= \cos(330^\circ) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

Problem 2

Use the half angle identity to find: $\sin\left(\frac{\pi}{8}\right)$

$$\begin{aligned} \sin\left(\frac{\pi}{8}\right) &= \sin\left(\frac{\frac{\pi}{4}}{2}\right) \\ &= \pm \sqrt{\frac{1 - \cos\left(\frac{\pi}{4}\right)}{2}} \\ &= \sqrt{\frac{1 - \frac{\sqrt{2}}{2}}{2} \cdot \frac{2}{2}} \\ &= \sqrt{\frac{2 - \sqrt{2}}{4}} \\ &= \frac{\sqrt{2 - \sqrt{2}}}{2} \end{aligned}$$

Note, since $\frac{\pi}{8}$ is in Q1 we only want the positive answer.

Problem 3

Find the exact value: $1 - 2 \sin^2(15^\circ)$

$$\begin{aligned} 1 - 2 \sin^2(15^\circ) &= \cos(2(15^\circ)) \\ &= \cos(30^\circ) \\ &= \frac{\sqrt{3}}{2} \end{aligned}$$

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Problem 4

Use power reducing formulas to find: $\sin^2(x) \cdot \cos^2(x)$

$$\begin{aligned}\sin^2(x) \cdot \cos^2(x) &= \left(\frac{1 - \cos(2x)}{2} \right) \left(\frac{1 + \cos(2x)}{2} \right) \\ &= \frac{1 + \cos(2x) - \cos(2x) - \cos^2(2x)}{4} \\ &= \frac{1}{4}(1 - \cos^2(2x)) \\ &= \frac{1}{4} \left(1 - \left(\frac{1 + \cos(4x)}{2} \right) \right) \\ &= \frac{1}{4} \left(1 - \frac{1}{2} - \frac{\cos(4x)}{2} \right) \\ &= \frac{1}{8} - \frac{\cos(4x)}{8} \\ &= \frac{1 - \cos(4x)}{8}\end{aligned}$$

Problem 5

Find $\tan\left(\frac{7\pi}{8}\right)$

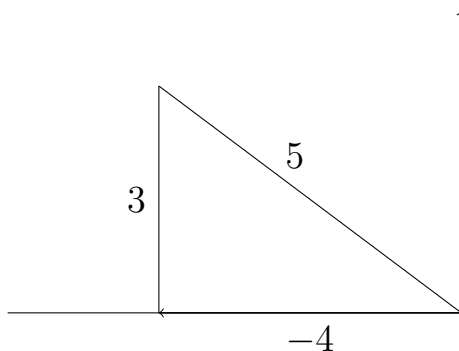
$$\begin{aligned}\tan\left(\frac{7\pi}{8}\right) &= \tan\left(\frac{7\pi}{4}\right) \\ &= \frac{1 - \cos\left(\frac{7\pi}{4}\right)}{\sin\left(\frac{7\pi}{4}\right)} \\ &= \frac{1 - \frac{\sqrt{2}}{2}}{-\frac{\sqrt{2}}{2}} \cdot \frac{2}{2} \\ &= \frac{2 - \sqrt{2}}{-\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{2\sqrt{2} - 2}{-2} \\ &= 1 - \sqrt{2}\end{aligned}$$

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Problem 6

Given $\sin(\theta) = \frac{3}{5}$, in Q2, find $\sin(2\theta)$, $\cos(2\theta)$, and $\tan(2\theta)$

Graph of θ



$$\begin{aligned}\sin(2\theta) &= 2 \sin(\theta) \cos(\theta) \\ &= 2 \left(\frac{3}{5}\right) \left(\frac{-4}{5}\right) \\ &= \frac{-24}{25}\end{aligned}$$

$$\begin{aligned}\cos(2\theta) &= \cos^2(\theta) - \sin^2(\theta) \\ &= \left(\frac{-4}{5}\right)^2 - \left(\frac{3}{5}\right)^2 \\ &= \frac{16}{25} - \frac{9}{25} \\ &= \frac{7}{25}\end{aligned}$$

$$\begin{aligned}\tan(2\theta) &= \frac{2 \tan(\theta)}{1 - \tan^2(\theta)} \\ &= \frac{2\left(\frac{3}{-4}\right)}{1 - \left(\frac{3}{-4}\right)^2} = \frac{-\frac{6}{4}}{1 - \frac{9}{16}} \\ &= \frac{-\frac{3}{2}}{\frac{7}{16}} = -\frac{3}{2} \cdot \frac{16}{7} = \frac{-48}{14} \\ &= \frac{-24}{7}\end{aligned}$$