Problem 1

Using the unit circle, find all the solutions of $\tan(x) = -1$ in the interval $[0, 2\pi)$

$$\tan(x) = -1$$

$$\tan(\frac{3\pi}{4}) = -1$$

$$\tan(\frac{7\pi}{4}) = -1$$

Thus, our solutions are $x = \frac{3\pi}{4}$ and $x = \frac{7\pi}{4}$

Problem 2

Find the two general solution equations for $\sin(x) = -\frac{1}{2}$

$$\sin(x) = -\frac{1}{2}$$

$$\alpha = \sin^{-1}(-\frac{1}{2}) = -\frac{\pi}{6}$$

$$x = \frac{-\pi}{6} + 2n\pi \text{ and } x = (\pi - \frac{-\pi}{6}) + 2n\pi$$

Simplifying the right equation, $\pi = \frac{6\pi}{6} \implies (\pi - \frac{-\pi}{6}) = (\frac{6\pi}{6} + \frac{\pi}{6}) = \frac{7\pi}{6}$

$$\implies x = \frac{-\pi}{6} + 2n\pi \text{ and } x = \frac{7\pi}{6} + 2n\pi$$

Problem 3

Find all the solutions in the interval $[0, 2\pi)$ of: $\sin(2x) = -\frac{\sqrt{3}}{2}$

$$\sin(2x) = -\frac{\sqrt{3}}{2}$$

$$\alpha = \sin^{-1}(-\frac{\sqrt{3}}{2}) = \frac{-\pi}{3}$$

$$2x = \frac{-\pi}{3} + 2n\pi \text{ and } 2x = (\pi - \frac{-\pi}{3}) + 2n\pi$$

$$2x = \frac{-\pi}{3} + 2n\pi \text{ and } 2x = \frac{3\pi}{3} + \frac{\pi}{3} + 2n\pi$$

$$2x = \frac{-\pi}{3} + 2n\pi \text{ and } 2x = \frac{4\pi}{3} + 2n\pi$$

We divide everything by 2

$$x = \frac{-\pi}{6} + n\pi \text{ and } x = \frac{4\pi}{6} + n\pi$$

Now we will find the solutions in the targeted interval

$$n = 0 : \text{First Equation } \frac{-\pi}{6}\text{✗} \quad \text{Second Equation } \frac{4\pi}{6} = \frac{2\pi}{3}\text{✓}$$

$$n = 1 : \text{First Equation } \frac{-\pi}{6}+\frac{6\pi}{6} = \frac{5\pi}{6}\text{✓} \quad \text{Second Equation } \frac{4\pi}{6}+\frac{6\pi}{6} = \frac{10\pi}{6} = \frac{5\pi}{3}\text{✓}$$

$$n = 2 : \text{First Equation } \frac{-\pi}{6}+\frac{12\pi}{6} = \frac{11\pi}{6}\text{✓} \quad \text{Second Equation } \frac{4\pi}{6}+\frac{12\pi}{6} = \frac{16\pi}{6}\text{✗}$$

$$x = \frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6} \text{ are the solutions}$$

Problem 4

Find the general solutions AND all the solutions on the interval $[0, 2\pi)$:

$(\cos(x) + 1)(\tan(x) + 1) = 0$

$$(\cos(x) + 1)(\tan(x) + 1) = 0$$

$$\cos(x) = -1 \text{ and } \tan(x) = -1$$

$$\cos(x) = -1 \implies \alpha = \cos^{-1}(-1) = \pi$$

$$x = \pi + 2n\pi \text{ and } x = (2\pi - \pi) + 2n\pi = \pi + 2n\pi$$

Note, the two equations are equal when simplified.

$$\tan(x) = -1 \implies \alpha = \tan^{-1}(-1) = \frac{-\pi}{4}$$

$$x = \frac{-\pi}{4} + n\pi$$

$$n = 0 : \text{First Equation } \pi\checkmark \quad \text{Second Equation } \frac{-\pi}{4}\ \text{✗}$$

$$n = 1 : \text{First Equation } \pi + 2\pi = 3\pi\text{✗} \quad \text{Second Equation } \frac{-\pi}{4} + \frac{4\pi}{4} = \frac{3\pi}{4}\checkmark$$

Note, for $n > 0$, only solutions to $\tan(x) + 1 = 0$ are in our interval.

$$n = 2 : \text{Second Equation } \frac{-\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}\checkmark$$

$$n = 3 : \text{Second Equation } \frac{-\pi}{4} + \frac{12\pi}{4} = \frac{11\pi}{4}\text{✗}$$

$$x = \pi, \frac{3\pi}{4}, \frac{7\pi}{4} \text{ are the solutions}$$

Problem 5

Find all the solutions in the interval $[0, 2\pi)$ of

$(2\cos(x)+1)(\sqrt{3}\tan(x)-1)=0$

$$(2\cos(x)+1)(\sqrt{3}\tan(x)-1)=0$$

$$\cos(x)=\frac{-1}{2} \implies \alpha=\cos^{-1}(\frac{-1}{2})=\frac{2\pi}{3}$$

$$\tan(x)=\frac{1}{\sqrt{3}} \implies \alpha=\tan^{-1}(\frac{1}{\sqrt{3}})=\frac{\pi}{6}$$

$$(2\cos(x)+1)=0 \implies x=\frac{2\pi}{3}+2n\pi \text{ and } x=(2\pi-\frac{2\pi}{3})+2n\pi=\frac{4\pi}{3}+2n\pi$$

$$(\sqrt{3}\tan(x)-1)=0 \implies x=\frac{\pi}{6}+n\pi$$

$n=0$: First Equation $\frac{2\pi}{3}$✓ Second Equation $\frac{4\pi}{3}$✓ Third Equation $\frac{\pi}{6}$✓

$n=1$: First Equation $\frac{2\pi}{3}+2\pi$✗ Second Equation $\frac{4\pi}{3}+2\pi$✗

Third Equation $\frac{\pi}{6}+\frac{6\pi}{6}=\frac{7\pi}{6}$✓

Both the first two equations will not work for $n>0$ so we will only check the third equation,

$n=2$: Third Equation $\frac{\pi}{6}+2\pi$✗

$x=\frac{\pi}{6},\frac{2\pi}{3},\frac{7\pi}{6},\frac{4\pi}{3}$ are the solutions

Problem 6

Find all the solutions in the interval $[0, 6\pi)$ of: $\sin(\frac{x}{2}) = \frac{\sqrt{2}}{2}$

$$\sin(\frac{x}{2}) = \frac{\sqrt{2}}{2}$$

$$\alpha = \sin^{-1}(\frac{\sqrt{2}}{2}) = \frac{\pi}{4}$$

$$\frac{x}{2} = \frac{\pi}{4} + 2n\pi \text{ and } \frac{x}{2} = (\pi - \frac{\pi}{4}) + 2n\pi = \frac{3\pi}{4} + 2n\pi$$

Multiplying everything by 2

$$x = \frac{\pi}{2} + 4n\pi \text{ and } x = \frac{3\pi}{2} + 4n\pi$$

$n = 0$: First Equation $\frac{\pi}{2}$✓ Second Equation $\frac{3\pi}{2}$✓

$n = 1$: First Equation $\frac{\pi}{2}+\frac{8\pi}{2} = \frac{9\pi}{2}$✓ Second Equation $\frac{3\pi}{2}+\frac{8\pi}{2} = \frac{11\pi}{2}$✓

$n = 2$: First Equation $\frac{\pi}{2}+\frac{16\pi}{2} = \frac{17\pi}{2}$ ✗ Second Equation $\frac{3\pi}{2}+\frac{16\pi}{2} = \frac{19\pi}{2}$ ✗

$$x = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{9\pi}{2}, \frac{11\pi}{2} \text{ are the solutions}$$