Problem 1

Solve and state the type of solution:

$$\begin{cases} y = -2x + 3 \\ y = 3x + 5 \end{cases}$$

Step 1: Set the Equations Equal to Each Other

Since both equations are equal to y, we set the right-hand sides of the equations equal to each other:

$$-2x + 3 = 3x + 5$$

Step 2: Solve for x

Combine like terms to solve for x:

$$-2x + 3 = 3x + 5$$

$$3 - 5 = 3x + 2x$$

$$-2 = 5x$$

$$x = -\frac{2}{5}$$

Step 3: Substitute x Back into One of the Original Equations to Find y

We substitute $x = -\frac{2}{5}$ into either of the original equations. Using $y = -2x + 3$:

$$y = -2\left(-\frac{2}{5}\right) + 3$$

$$y = \frac{4}{5} + 3$$

$$y = \frac{4}{5} + \frac{15}{5}$$
$$y = \frac{19}{5}$$

Step 4: State the Solution

The solution to the system is:

$$(x, y) = \left(-\frac{2}{5}, \frac{19}{5}\right)$$

Step 5: Determine the Type of Solution

Since the system has exactly one solution, it is a **consistent independent** system.

Problem 2

Solve and state the type of solution:

$$\begin{cases} x - 2y = 15 \\ 2x + 4y = -18 \end{cases}$$

Step 1: Simplify the Second Equation

First, simplify the second equation by dividing all terms by 2:

$$2x + 4y = -18 \implies x + 2y = -9$$

Now we have the simplified system:

$$\begin{cases} x - 2y = 15 \\ x + 2y = -9 \end{cases}$$

Step 2: Subtract the Equations

Subtract the second equation from the first equation to eliminate x:

$$(x - 2y) - (x + 2y) = 15 - (-9)$$

$$x - 2y - x - 2y = 15 + 9$$

$$-4y = 24$$

$$y = -6$$

Step 3: Substitute y Back into One of the Original Equations to Find x

We substitute $y = -6$ into the first original equation:

$$x - 2(-6) = 15$$

$$x + 12 = 15$$

$$x = 3$$

Step 4: State the Solution

The solution to the system is:

$$(x, y) = (3, -6)$$

Step 5: Determine the Type of Solution

Since the system has exactly one solution, it is a **consistent independent** system.

Problem 3

Solve and state the type of solution:

$$\begin{cases} 6x - 2y = 10 \\ 2x + 2y = 14 \end{cases}$$

Step 1: Simplify the Second Equation

First, simplify the second equation by dividing all terms by 2:

$$2x + 2y = 14 \implies x + y = 7$$

Now we have the simplified system:

$$\begin{cases} 6x - 2y = 10 \\ x + y = 7 \end{cases}$$

Step 2: Solve the Simplified System Using the Substitution Method

Solve the second equation for y:

$$x + y = 7 \implies y = 7 - x$$

Substitute $y = 7 - x$ into the first equation:

$$6x - 2(7 - x) = 10$$

Step 3: Simplify and Solve for x

Simplify the equation:

$$6x - 14 + 2x = 10$$

$$8x - 14 = 10$$

$$8x = 24$$

$$x = 3$$

Step 4: Substitute x Back into One of the Original Equations to Find y

We substitute $x = 3$ into the equation $y = 7 - x$:

$$y = 7 - 3$$

$$y = 4$$

Step 5: State the Solution

The solution to the system is:

$$(x, y) = (3, 4)$$

Step 6: Determine the Type of Solution

Since the system has exactly one solution, it is a **consistent independent** system.

Problem 4

Solve and state the type of solution:

$$\begin{cases} 3x - y = 2 \\ 9x - 3y = 6 \end{cases}$$

Step 1: Simplify the Second Equation

First, simplify the second equation by dividing all terms by 3:

$$9x - 3y = 6 \implies 3x - y = 2$$

Now we have the simplified system:

$$\begin{cases} 3x - y = 2 \\ 3x - y = 2 \end{cases}$$

Step 2: Analyze the Simplified System

Notice that both equations are identical:

$$3x - y = 2$$

This means that both equations represent the same line. Therefore, every solution of one equation is also a solution of the other.

Step 3: State the Solution

Since the two equations represent the same line, the system has infinitely many solutions. The solutions can be expressed in the form:

$$(x, y) = (t, 3t - 2) \quad \text{for any real number } t$$

Step 4: Determine the Type of Solution

Since the system has infinitely many solutions, it is a **consistent dependent** system.

Problem 5

Solve and state the type of solution:

$$\begin{cases} 2x - 3y = 5 \\ 6x - 9y = 10 \end{cases}$$

Step 1: Simplify the Second Equation

First, simplify the second equation by dividing all terms by 3:

$$6x - 9y = 10 \implies 2x - 3y = \frac{10}{3}$$

Now we have the simplified system:

$$\begin{cases} 2x - 3y = 5 \\ 2x - 3y = \frac{10}{3} \end{cases}$$

Step 2: Analyze the Simplified System

Notice that the two equations have the same left-hand side but different right-hand sides:

$$2x - 3y = 5$$
$$2x - 3y = \frac{10}{3}$$

This means that the equations are parallel lines and do not intersect.

Step 3: State the Solution

Since the two equations represent parallel lines that do not intersect, the system has no solution.

Step 4: Determine the Type of Solution

Since the system has no solution, it is an **inconsistent** system.

Problem 6

Solve and state the type of solution:

$$\begin{cases} \frac{1}{3}x + y = 6 \\ \frac{1}{2}x - \frac{1}{4}y = 2 \end{cases}$$

Step 1: Eliminate the Fractions

To eliminate the fractions, multiply each equation by the least common multiple (LCM) of the denominators.

For the first equation, multiply by 3:

$$3\left(\frac{1}{3}x + y\right) = 3 \cdot 6 \implies x + 3y = 18$$

For the second equation, multiply by 4:

$$4\left(\frac{1}{2}x - \frac{1}{4}y\right) = 4 \cdot 2 \implies 2x - y = 8$$

Now the system is:

$$\begin{cases} x + 3y = 18 \\ 2x - y = 8 \end{cases}$$

Step 2: Eliminate y

To eliminate y, multiply the first equation by 1 and the second equation by 3:

$$1(x + 3y) = 1 \cdot 18 \implies x + 3y = 18$$

$$3(2x - y) = 3 \cdot 8 \implies 6x - 3y = 24$$

Now we have:

$$\begin{cases} x + 3y = 18 \\ 6x - 3y = 24 \end{cases}$$

Add the equations to eliminate y:

$$(x + 3y) + (6x - 3y) = 18 + 24 \implies 7x = 42$$

$$x = 6$$

Step 3: Solve for y

Substitute $x = 6$ into the first simplified equation:

$$6 + 3y = 18$$

$$3y = 12$$

$$y = 4$$

Step 4: State the Solution

The solution to the system is:

$$(x, y) = (6, 4)$$

Step 5: Determine the Type of Solution

Since the system has exactly one solution, it is a **consistent independent** system.

Problem 7

Solve and state the type of solution:

$$\begin{cases} 0.3x + 0.2y = 4 \\ 0.5x - 0.4y = 0.7 \end{cases}$$

Step 1: Clear the Fractions

Multiply both equations by 10 to eliminate the decimals:

$$\begin{cases} 3x + 2y = 40 \\ 5x - 4y = 7 \end{cases}$$

Step 2: Solve the System Using the Elimination Method

Multiply the first equation by 2 to make the coefficients of y equal:

$$\begin{cases} 6x + 4y = 80 \\ 5x - 4y = 7 \end{cases}$$

Add the two equations to eliminate y:

$$6x + 4y + 5x - 4y = 80 + 7$$

$$11x = 87$$

Solve for x:

$$x = \frac{87}{11}$$

Substitute $x = \frac{87}{11}$ back into the first equation to solve for y:

$$3\left(\frac{87}{11}\right) + 2y = 40$$

$$\frac{261}{11} + 2y = 40$$

$$2y = 40 - \frac{261}{11}$$
$$2y = \frac{440}{11} - \frac{261}{11}$$
$$2y = \frac{179}{11}$$
$$y = \frac{179}{22}$$

Step 3: Determine the Type of Solution

The solution to the system of equations is:

$$(x, y) = \left(\frac{87}{11}, \frac{179}{22}\right)$$

Since the system has exactly one solution, it is a **consistent independent** system.