Problem 1

Identify all of the coefficients (signify which one is the leading coefficient), and degree of the polynomial $5x^2 - 7x^3 + 8x + 2$

Coefficients: 5, -7, 8, 2

Leading Coefficient: -7

Degree: 3

Problem 2

Perform the indicated operation: $(5x + 3)^2$

Write out the expression as multiplying the expression with itself, and then apply the FOIL Method.

$$\begin{aligned}(5x+3)^2 &= (5x+3)(5x+3)\\ &= 25x^2 + 15x + 15x + 9\\ &= 25x^2 + 30x + 9\end{aligned}$$

Problem 3

Find and simplify the product: $(x^2 + 3x + 2)(x + 4)$

Use the FOIL Method to find the product.

$$\begin{aligned}(x^2+3x+2)(x+4) &= x^3 + 4x^2 + 3x^2 + 12x + 2x + 8\\ &= x^3 + 7x^2 + 14x + 8\end{aligned}$$

Problem 4

Factor the polynomial: $x^2 - 12x + 32$

Since the leading coefficient is 1, the two expressions will start with x.

$$(x + \underline{\quad\quad})(x + \underline{\quad\quad})$$

Now we need to see which two numbers multiply to 32 and adds together to get -12. The pair is -4 and -8. So, our factorization of the original expression would be:

$$(x + (-4))(x + (-8)) = (x - 4)(x - 8)$$

Problem 5

Factor the polynomial: $2x^2 - 7x + 3$

To factor $2x^2 - 7x + 3$, we use the AC Method and look for two numbers that multiply to $AC = 2 \cdot 3 = 6$ and add up to -7. The numbers that satisfy this condition are -6 and -1. So, we can rewrite the expression as:

$$2x^2 - 6x - 1x + 3$$

Now, we group the terms (aka cut the polynomial in half):

$$(2x^2 - 6x) + (-x + 3)$$

Now, let's factor out the common factors from each group (aka factor the left and factor the right):

$$2x(x - 3) - 1(x - 3)$$

Notice, that we have a common factor of $x - 3$, so factor it out and write the leftovers to get the final answer

$$(x - 3)(2x - 1)$$

Problem 6

Factor the polynomial: $3x^2 - 5x - 2$

To factor $3x^2 - 5x - 2$, we use the AC method again and look for two numbers that multiply to $AC = 3 \cdot -2 = -6$ and add up to -5. The numbers that satisfy this condition are -6 and 1. So, we can rewrite the expression as:

$$3x^2 - 6x + x - 2$$

Now, we group the terms (aka cut the polynomial in half):

$$(3x^2 - 6x) + (x - 2)$$

Now, let's factor out the common factors from each group (aka factor the left and factor the right):

$$3x(x - 2) + 1(x - 2)$$

Notice, that we have a common factor of $x - 2$, so factor it out and write the leftovers to get the final answer

$$(x - 2)(3x + 1)$$

Problem 7

Factor the polynomial: $x^3 - 5x^2 + 4x - 20$

We first need to divide the polynomial into groups (aka cut in half) and then factor out a GCF in each group (aka factor the left and factor the right).

$$\begin{aligned} x^3 - 5x^2 + 4x - 20 &= (x^3 - 5x^2) + (4x - 20) \\ &= x^2(x - 5) + 4(x - 5) \\ &= (x - 5)(x^2 + 4) \end{aligned}$$

Problem 8

Is the following a polynomial: $x^{-2} + 3x + 5$?

No, you can't have a negative exponent in a polynomial.

Problem 9

Simplify: $(5x^2 + 3x - 1) - (x^2 - 2x + 3) + (2x^2 + x + 5)$

Distribute and gather like terms,

$$\begin{aligned} &= (5x^2 + 3x - 1) - (x^2 - 2x + 3) + (2x^2 + x + 5) \\ &= 5x^2 + 3x - 1 - x^2 + 2x - 3 + 2x^2 + x + 5 \\ &= 6x^2 + 6x + 1 \end{aligned}$$

Problem 10

What is the coefficient of the x term in the product $(3x - 5)(x + 2)$

First FOIL, and then find the coefficient (number in front) of x,

$$\begin{aligned} &= (3x - 5)(x + 2) \\ &= 3x^2 + 6x - 5x - 10 \\ &= 3x^2 + x - 10 \end{aligned}$$

So the coefficient is 1.

Problem 11

When the leading coefficient of a trinomial is not 1, which factoring method do you use?

AC Method

Problem 12

Factor the polynomial: $x^2 - 9$

This is a special case called difference of squares. What times what is "x" and what times what is "9", remember when FOILing the "o" and the "i" cancel out, so we need to have one "+" and one "-"

$$(x + 3)(x - 3)$$