Activity A.8 - Answer Key

Problem 1

What is i^{11}?

We know that $i^4 = 1$. Therefore, we can rewrite i^{11} as:

$$i^{11} = i^{4\cdot 2+3}$$

Using the properties of exponents, we can rewrite this as:

$$i^{11} = (i^4)^2 \cdot i^3$$

Since $i^4 = 1$, this simplifies to:

$$i^{11} = (1)^2 \cdot i^3$$

$$i^{11} = i^3$$

Now, i^3 can be rewritten as $i^2 \cdot i$. Since $i^2 = -1$, we have:

$$i^{11} = (-1) \cdot i$$

Simplifying further, we get:

$$i^{11} = -i$$

Thus, $i^{11} = -i$.

Problem 2

What is i^{27}?

We know that $i^4 = 1$. Therefore, we can rewrite i^{27} as:

$$i^{27} = (i^4)^6 \cdot i^3$$

Using the properties of exponents, we can simplify this expression:

$$i^{27} = (1)^6 \cdot i^3$$

$$i^{27} = i^3$$

Now, i^3 can be rewritten as $i^2 \cdot i$. Since $i^2 = -1$, we have:

$$i^{27} = (-1) \cdot i$$

Simplifying further, we get:

$$i^{27} = -i$$

Thus, $i^{27} = -i$.

Problem 3

Simplify the expression: $(6 - 2i)(2 - 3i)$

We expand the expression using the distributive property:

$$(6 - 2i)(2 - 3i) = 6(2) + 6(-3i) - 2i(2) - 2i(-3i)$$

Simplify each term:

$$= 12 - 18i - 4i + 6i^2$$

Remember that $i^2 = -1$:

$$= 12 - 18i - 4i + 6(-1)$$

$$= 12 - 18i - 4i - 6$$

Combine like terms:

$$= 6 - 22i$$

Thus, the simplified expression is:

$$\boxed{6 - 22i}$$

Problem 4

Simplify the expression: $(5-4i)-(7+8i)+(6-2i)$

First, distribute the negative sign in the second term:

$$(5-4i)-(7+8i)+(6-2i)=5-4i-7-8i+6-2i$$

Combine like terms:

$$=(5-7+6)+(-4i-8i-2i)$$

$$=4-14i$$

Thus, the simplified expression is:

$$\boxed{4-14i}$$

Problem 5

What is the conjugate of $8-7i$?

Our original equation is in the form of $a+bi$, where $a=8$ and $b=-7$. The conjugate of this would be in the form $a-bi$, so the conjugate of this equation is $8+7i$.

Problem 6

Write this quotient in standard form: $\frac{2}{i^{-5}}$

We start with the given expression:

$$\frac{2}{i^{-5}}$$

First, simplify the denominator using the property of exponents $i^{-5}=\frac{1}{i^5}$:

$$\frac{2}{i^{-5}}=2\cdot i^5$$

Next, recall the powers of i:

$$i = i, \quad i^2 = -1, \quad i^3 = -i, \quad i^4 = 1$$

Using $i^4 = 1$, we can simplify i^5:

$$i^5 = i^4 \cdot i = 1 \cdot i = i$$

So, we have:

$$2 \cdot i^5 = 2 \cdot i$$

Thus, the expression in standard form is:

$$\boxed{2i}$$

Problem 7

Write this quotient in standard form: $\frac{2-3i}{4-2i}$

We start with the given expression:

$$\frac{2-3i}{4-2i}$$

To write this in standard form, we multiply the numerator and the denominator by the complex conjugate of the denominator, which is $4+2i$:

$$\frac{2-3i}{4-2i} \cdot \frac{4+2i}{4+2i}$$

Simplify the numerator using the distributive property:

$$(2-3i)(4+2i) = 2 \cdot 4 + 2 \cdot 2i - 3i \cdot 4 - 3i \cdot 2i$$

$$= 8 + 4i - 12i - 6i^2$$

Since $i^2 = -1$:

$$= 8 + 4i - 12i - 6(-1)$$

$$= 8 + 4i - 12i + 6$$

$$= 14 - 8i$$

Next, simplify the denominator using the difference of squares formula:

$$(4 - 2i)(4 + 2i) = 4^2 - (2i)^2$$

$$= 16 - 4i^2$$

Since $i^2 = -1$:

$$= 16 - 4(-1)$$

$$= 16 + 4$$

$$= 20$$

So, we have:

$$\frac{2 - 3i}{4 - 2i} = \frac{14 - 8i}{20}$$

Separate the real and imaginary parts:

$$= \frac{14}{20} - \frac{8i}{20}$$

Simplify each part:

$$= \frac{7}{10} - \frac{2i}{5}$$

Thus, the expression in standard form is:

$$\boxed{\frac{7}{10} - \frac{2i}{5}}$$

Problem 8

Write this quotient in standard form: $\frac{-3}{i}$

To write the quotient $\frac{-3}{i}$ in standard form, we need to eliminate the imaginary unit i from the denominator. We can do this by multiplying the numerator and the denominator by the complex conjugate of the denominator, which in this case is $-i$:

Multiply the numerator and the denominator by $-i$:

$$\frac{-3}{i} \cdot \frac{-i}{-i} = \frac{-3 \cdot (-i)}{i \cdot (-i)}$$

Simplify the numerator:

$$-3 \cdot (-i) = 3i$$

Simplify the denominator:

$$i \cdot (-i) = -i^2$$

Since $i^2 = -1$, we have:

$$-i^2 = -(-1) = 1$$

Combine the simplified numerator and denominator:

$$\frac{3i}{1} = 3i$$

So, the quotient $\frac{-3}{i}$ in standard form is $3i$.

Problem 9

Write this quotient in standard form: $\frac{2-i}{5+i}$

To write the quotient $\frac{2-i}{5+i}$ in standard form, we need to eliminate the imaginary unit from the denominator. We do this by multiplying both the numerator and the denominator by the complex conjugate of the denominator, which is $5 - i$:

Multiply the numerator and the denominator by the complex conjugate of the denominator:

$$\frac{2-i}{5+i} \cdot \frac{5-i}{5-i} = \frac{(2-i)(5-i)}{(5+i)(5-i)}$$

Simplify the denominator:

$$(5+i)(5-i) = 5^2 - i^2 = 25 - (-1) = 25 + 1 = 26$$

Expand the numerator:

$$(2-i)(5-i) = 2 \cdot 5 + 2 \cdot (-i) - i \cdot 5 - i \cdot (-i)$$

$$= 10 - 2i - 5i + i^2$$

Since $i^2 = -1$, this becomes:

$$= 10 - 2i - 5i - 1 = 10 - 7i - 1 = 9 - 7i$$

Combine the results:

$$\frac{2-i}{5+i} = \frac{9-7i}{26}$$

Separate the real and imaginary parts:

$$\frac{9-7i}{26} = \frac{9}{26} - \frac{7i}{26}$$

So, the quotient $\frac{2-i}{5+i}$ in standard form is $\frac{9}{26} - \frac{7}{26}i$.