Activity Exam #1 Review - Answer Key

Problem 1

True of False: The center of the circle given by $(x+3)^2 + (y+4)^2 = 9$ is the point $(3, 4)$.

False. The actual center for the circle would be $(-3, -4)$ since the standard form of the equation of a circle is $(x-a)^2 + (y-b)^2 = r^2$, with (a, b) being the center of the circle.

Problem 2

True of False: The graph of the line $x = -5$ is a vertical line.

True.

Problem 3

True of False: In the function notation $y = f(x)$, x is the output.

False. x would be the input and y would be the output here.

Problem 4

True of False: The average rate of change of an increasing function is negative.

False. For an increasing function, its rate of change would be positive. Thus, with only a positive rate of change for the points where the function is increasing, its average rate of change would also be positive, not negative.

Problem 5

True of False: The function $f(x) = (\frac{1}{2}x)^2$ has a horizontal compression.

False. This would result in a horizontal stretch, since we are multiplying x by a number between 0 and 1. If we multiplied x by something greater than 1, it would result in a horizontal compression.

Problem 6

True of False: If a function is one-to-one, then its inverse exists.

True.

Problem 7

Fill in the blank: The standard form of the equation of a circle with center (h, k) and radius r is: ____________

$(x-h)^2 + (y-k)^2 = r^2$

Problem 8

Fill in the blank: Every line parallel to the line $y = 3x - 2$ has a slope equal to ____________.

3. This is because parallel lines have the same slope as each other.

Problem 9

Fill in the blank: The average rate of change of f as x changes from a to b is ____________.

$\frac{f(b)-f(a)}{b-a}$

Problem 10

Fill in the blank: A function is even if $f(-x) =$ ____________.

$f(x)$ for all x in the domain.

Problem 11

Fill in the blank: The graph of $y = f(x+3)$ is found by shifting the graph of $y = f(x)$ three units to the ____________.

Left. When we have $y = f(x+c)$, this represents a horizontal shift of the graph of $y = f(x)$:

If c is positive, the graph shifts to the left by c units. If c is negative, the graph shifts to the right by $|c|$ units.

In our case, $c = 3$, which is positive. This means the graph of $y = f(x+3)$ is shifted to the left by 3 units.

Problem 12

Fill in the blank: To calculate $(f \circ g)(x)$, I plug ________ into ________.

$g(x)$ into $f(x)$.

Problem 13

Fill in the blank: A consistent system of equations has a ____________.

solution (will also accept if it has at least one solution).

Problem 14

Find the difference quotient of $f(x) = x^2 - 3x$.

Step 1: Compute $f(x+h)$

First, substitute $x + h$ into the function $f(x)$:

$$f(x+h) = (x+h)^2 - 3(x+h)$$

Expand and simplify:

$$f(x+h) = (x^2 + 2xh + h^2) - 3(x+h)$$

$$f(x+h) = x^2 + 2xh + h^2 - 3x - 3h$$

Step 2: Compute $f(x+h) - f(x)$

Now, subtract $f(x)$ from $f(x+h)$:

$$f(x+h) - f(x) = (x^2 + 2xh + h^2 - 3x - 3h) - (x^2 - 3x)$$

Simplify the expression:

$$f(x+h) - f(x) = x^2 + 2xh + h^2 - 3x - 3h - x^2 + 3x$$

$$f(x+h) - f(x) = 2xh + h^2 - 3h$$

Step 3: Divide by h

Divide the simplified expression by h:

$$\frac{f(x+h) - f(x)}{h} = \frac{2xh + h^2 - 3h}{h}$$

Simplify the fraction:

$$\frac{f(x+h) - f(x)}{h} = \frac{h(2x + h - 3)}{h}$$

$$\frac{f(x+h) - f(x)}{h} = 2x + h - 3$$

Conclusion

The difference quotient of $f(x) = x^2 - 3x$ is:

$$\boxed{2x + h - 3}$$

Problem 15

State the distance formula.

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Problem 16

State the midpoint formula.

Midpoint= $(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2})$

Problem 17

State the average rate of change formula.

Average rate of change $= \frac{f(x_2)-f(x_1)}{x_2-x_1}$

Problem 18

Find the center and radius of the circle: $x^2 + y^2 + 2x - 4y - 5 = 0$

Step 1: Rearrange the Equation

Group the x terms and y terms together:

$$x^2 + 2x + y^2 - 4y = 5$$

Step 2: Complete the Square

$$x^2 + 2x + (\frac{2}{2})^2 + y^2 - 4y + (\frac{-4}{2})^2 = 5 + (\frac{2}{2})^2 + (\frac{-4}{2})^2$$

$$x^2 + 2x + 1 + y^2 - 4y + 4 = 5 + 1 + 4$$

Step 3: Rewrite the Equation with Completed Squares

$$(x + 1)^2 + (y - 2)^2 = 10$$

Step 4: Identify the Center and Radius

The standard form of the equation of a circle is:

$$(x-h)^2 + (y-k)^2 = r^2$$

Comparing this with the equation $(x+1)^2 + (y-2)^2 = 10$, we get:

$$h = -1, \quad k = 2, \quad r^2 = 10$$

So, the center of the circle is:

$$\text{Center} = (-1, 2)$$

And the radius is:

$$r = \sqrt{10}$$

Conclusion

The center of the circle is:

$$\boxed{(-1, 2)}$$

The radius of the circle is:

$$\boxed{\sqrt{10}}$$

Problem 19

Is $f(x) = x^2 + x^4$, even, odd or neither?

Step 1: Define the Function

Given the function:

$$f(x) = x^2 + x^4$$

Step 2: Find $f(-x)$

Substitute $-x$ into the function:

$$f(-x) = (-x)^2 + (-x)^4$$

$$f(-x) = x^2 + x^4$$

Step 3: Compare $f(x)$ and $f(-x)$

We have:

$$f(x) = x^2 + x^4$$

$$f(-x) = x^2 + x^4$$

Since $f(x) = f(-x)$, the function is even.

Conclusion

The function $f(x) = x^2 + x^4$ is:

$$\boxed{\text{even}}$$

Problem 20

Given $f(x) = 2x + 1$ and $g(x) = 3x - 5$, find $(f \circ g)(x)$.

Step 1: Define the Composition

The composition of f and g is given by:

$$(f \circ g)(x) = f(g(x))$$

Step 2: Substitute $g(x)$ into $f(x)$

Substitute $g(x)$ into $f(x)$:

$$(f \circ g)(x) = f(3x - 5)$$

Since $f(x) = 2x + 1$, replace x in $f(x)$ with $3x - 5$:

$$f(3x - 5) = 2(3x - 5) + 1$$

Step 3: Simplify the Expression

Simplify the expression:

$$f(3x - 5) = 2 \cdot 3x - 2 \cdot 5 + 1$$

$$f(3x - 5) = 6x - 10 + 1$$

$$f(3x - 5) = 6x - 9$$

Conclusion

The composition $(f \circ g)(x)$ is:

$$(f \circ g)(x) = 6x - 9$$