Problem 1

True or False: All solutions of $\tan(x) = -1$ are given by the general solution equation: $\frac{-\pi}{4} + 2n\pi$

False.

The solutions for $\tan(x)$ are in the form "$stuff$" $= \alpha + n\pi$

The solution equation is $x = \dfrac{-\pi}{4} + n\pi$

Problem 2

Verify: $\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{\sin(\theta)}{1-\cos(\theta)} = 2\csc(\theta)$

Simplifying the left hand side of the equation, we get:

$$\frac{\sin(\theta)}{1+\cos(\theta)} + \frac{\sin(\theta)}{1-\cos(\theta)}$$

$$= \frac{\sin(\theta)(1-\cos(\theta))}{(1+\cos(\theta))(1-\cos(\theta))} + \frac{\sin(\theta)(1+\cos(\theta))}{(1-\cos(\theta))(1+\cos(\theta))}$$

$$= \frac{\sin(\theta) - \cos(\theta)\sin(\theta)}{1-\cos^2(\theta)} + \frac{\sin(\theta) + \cos(\theta)\sin(\theta)}{1-\cos^2(\theta)}$$

$$= \frac{\sin(\theta) - \cos(\theta)\sin(\theta)}{\sin^2(\theta)} + \frac{\sin(\theta) + \cos(\theta)\sin(\theta)}{\sin^2(\theta)}$$

$$= \frac{2\sin(\theta)}{\sin^2(\theta)}$$

$$= \frac{2}{\sin(\theta)}$$

$$= 2\csc(\theta)$$

Problem 3

Use the half-angle formula to find $\cos(15^\circ)$

$$\cos(\frac{\theta}{2}) = \pm\sqrt{\frac{1+\cos(\theta)}{2}}$$

$$\frac{\theta}{2} = 15^\circ \implies \theta = 30^\circ$$

$$\implies \cos(15^\circ) = \pm\sqrt{\frac{1+\cos(30^\circ)}{2}}$$

Note that 15° is in Q1 so $\cos(15^\circ)$ is positive.

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2}}$$

$$= \sqrt{\frac{1+\frac{\sqrt{3}}{2}}{2} \cdot \frac{2}{2}} = \sqrt{\frac{2+\sqrt{3}}{4}}$$

$$= \frac{\sqrt{2+\sqrt{3}}}{2}$$

Problem 4

Find the exact value of: $\cos(50^\circ)\cos(10^\circ) - \sin(50^\circ)\sin(10^\circ)$

$$\cos(50^\circ)\cos(10^\circ) - \sin(50^\circ)\sin(10^\circ)$$

$$= \cos(50^\circ + 10^\circ)$$

$$= \cos(60^\circ)$$

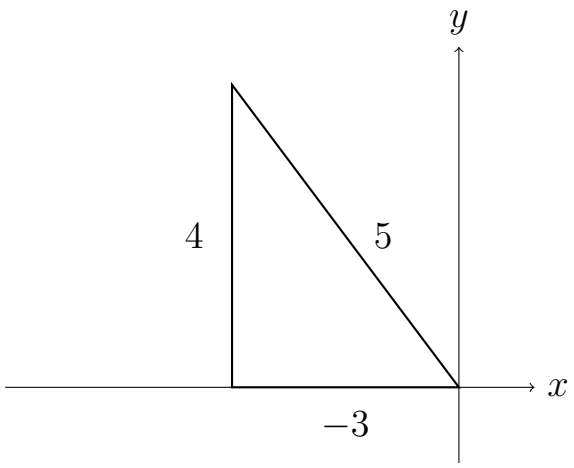
$$= \frac{1}{2}$$

Problem 5

Let $\sin(\theta) = \frac{4}{5}$ in Q2, and $\cos(\alpha) = \frac{5}{13}$ in Q4, find $\cos(\theta + \alpha)$

First we will find the values of sin and cos for θ

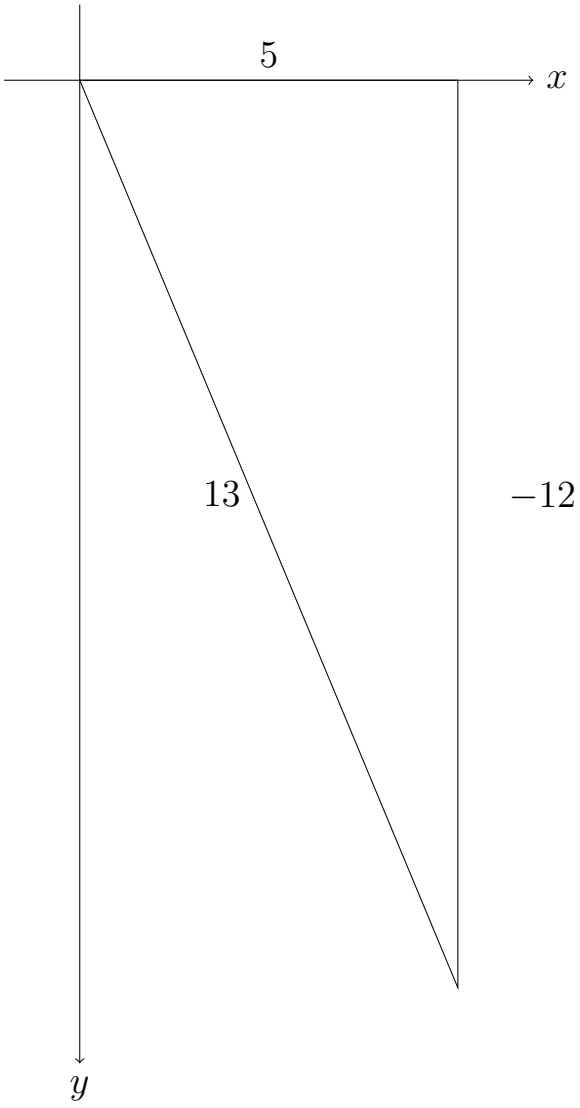
Graph of θ

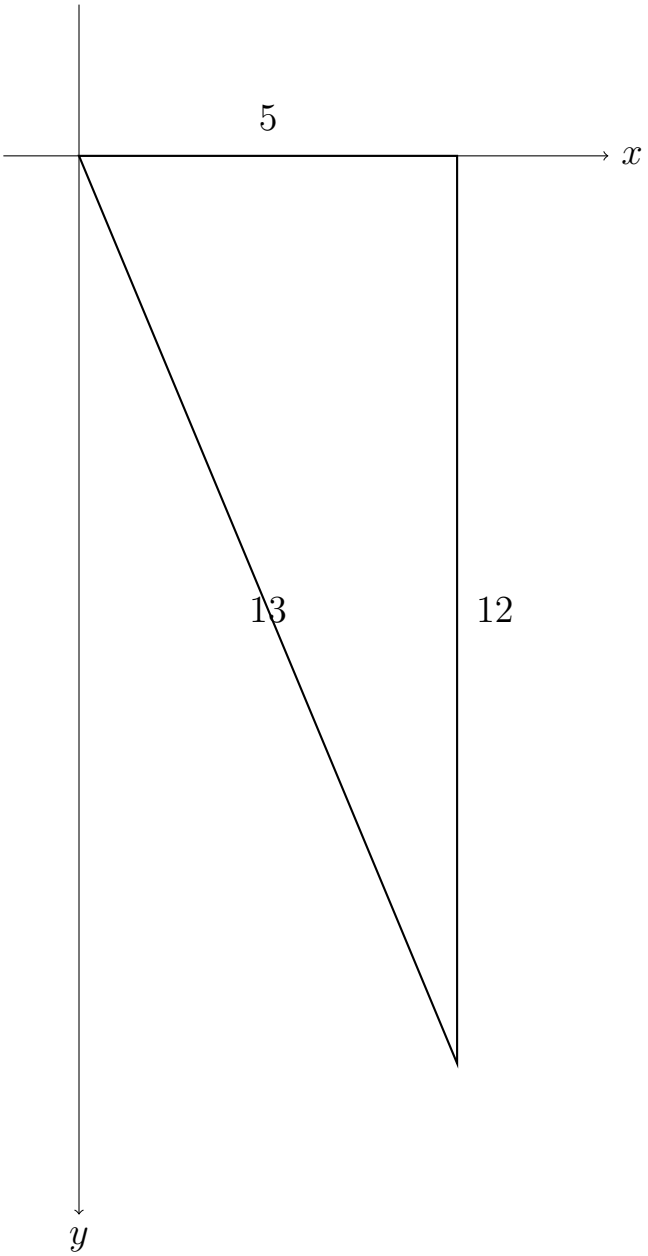


$$\implies \cos(\theta) = \frac{-3}{5}$$

Now we will find the values of sin and cos for α

Graph of α


5
x
13
−12
y



$$\implies \sin(\alpha) = \frac{-12}{13}$$

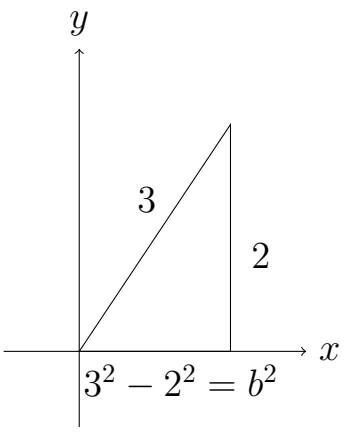
$$\implies \cos(\theta + \alpha) = (\frac{-3}{5})(\frac{5}{13}) - (\frac{4}{5})(\frac{-12}{13})$$
$$= \frac{-15}{65} - \frac{-48}{65} = \frac{33}{65}$$

Problem 6

Given $\sin(\theta) = \frac{2}{3}$ in Q1, find $\sin(2\theta)$

$$\sin(2\theta) = 2\sin(\theta)(\cos(\theta))$$

Graph of θ



$$\implies \sin(2\theta) = 2(\frac{2}{3})(\frac{b}{3}$$

$$= 2\frac{2}{3}\frac{\sqrt{5}}{3}$$

$$= \frac{4\sqrt{5}}{9}$$

Problem 7

Find the general solution equations for: $2\sin(\frac{1}{2}x - \frac{\pi}{4}) + \sqrt{3} = 0$

Problem 8

Find the general solutions and all the solutions in the interval $[0, 2\pi]$ for: $(\sqrt{3}\tan(\theta) + 1)(2\cos(\theta) + 1) = 0$