Problem 1

Find the difference quotient of $f(x) = -x^2 - x$

Given the function $f(x) = -x^2 - x$, the difference quotient is calculated as follows:

$$\text{Difference Quotient} = \frac{f(x+h) - f(x)}{h}$$

First, we need to find $f(x+h)$:

$$f(x+h) = -(x+h)^2 - (x+h)$$

Expanding $f(x+h)$:

$$f(x+h) = -(x^2 + 2xh + h^2) - x - h = -x^2 - 2xh - h^2 - x - h$$

Now, calculate the difference $f(x+h) - f(x)$:

$$f(x+h) - f(x) = (-x^2 - 2xh - h^2 - x - h) - (-x^2 - x)$$

Simplify the expression:

$$f(x+h) - f(x) = -x^2 - 2xh - h^2 - x - h + x^2 + x$$

Combine like terms:

$$f(x+h) - f(x) = -2xh - h^2 - h$$

Now, divide by h to obtain the difference quotient:

$$\frac{f(x+h) - f(x)}{h} = \frac{-2xh - h^2 - h}{h}$$

Simplify the fraction by dividing each term by h:

$$\frac{f(x+h) - f(x)}{h} = -2x - h - 1$$

Therefore, the difference quotient of the function is:

$$\boxed{-2x - h - 1}$$

Problem 2

Graph the function $f(x) = x(x-3)^2(x+2)^2$

Degree: $1 + 2 + 2 = 5$

Leading Coefficient: $1(1)^2(1)^2 = 1$

End Behavior: down, up

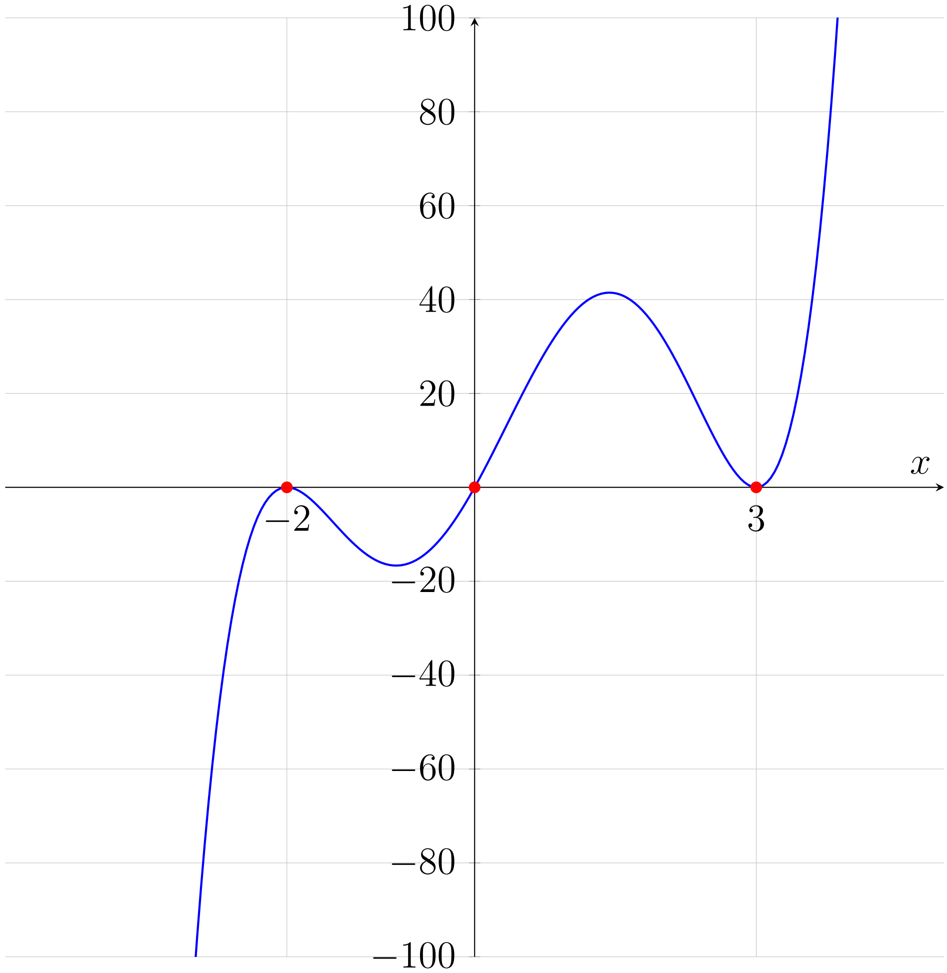
Zeros (in order left to right): $0, 3, -2$

Multiplicities (in order left to right): 1, 2, and 2

Cross/Touch (in order left to right): cross, touch, touch

y-intercept: $y = 0$

NLT: - - + +



Problem 3

Sketch 1 cycle of the graph $f(x) = -3\sin(\frac{1}{2}x) + 1$

Step 1:

$$\text{Amplitude} = |-3| = 3$$

$$\text{Period} = \frac{2\pi}{b} = \frac{2\pi}{\frac{1}{2}} = 4\pi$$

$$\text{Phase shift} = \text{c} = 0$$

$$\text{Vertical shift} = \text{d} = 1 \text{ (up)}$$

Step 2: Find the domain of 1 cycle using $[c, c + \frac{2\pi}{b}]$.

$$[0, 0 + 4\pi]$$

$$[0, 4\pi]$$

Step 3: Determine the key points,

$$\frac{1}{4}(4\pi) = \pi$$

$$0 + \pi = \pi,$$

$$\pi + \pi = 2\pi,$$

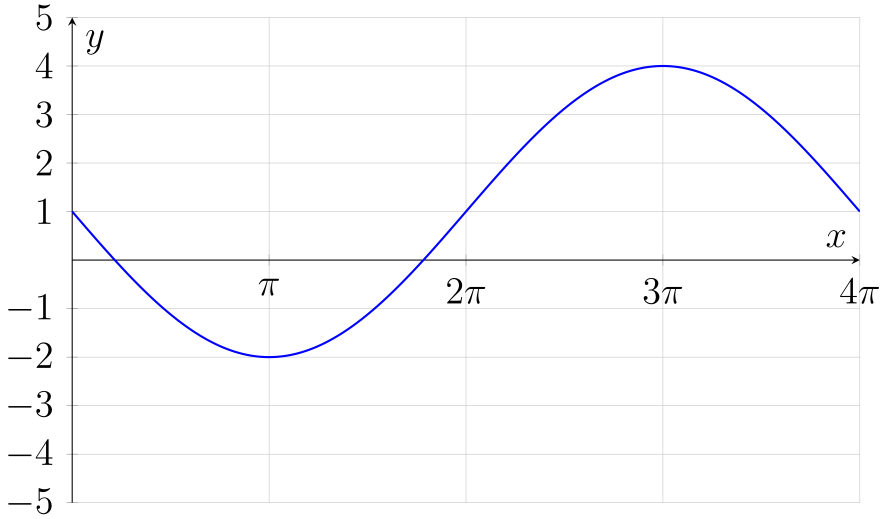
$$2\pi + \pi = 3\pi,$$

$$3\pi + \pi = 4\pi$$

$$\text{Key points} : 0, \pi, 2\pi, 3\pi, 4\pi$$

Step 4: X/Y chart and graph $\rightarrow$

X	Y
0	1
π	-2
2π	1
3π	4
4π	1



Problem 4

Find the inverse of $f(x) = \dfrac{3x+2}{5x-7}$

To find the inverse of the function $f(x) = \frac{3x+2}{5x-7}$, we follow these steps:

1. Replace $f(x)$ with y:

$$y = \frac{3x+2}{5x-7}$$

2. Switch x and y:

$$x = \frac{3y+2}{5y-7}$$

3. Solve for y:

Multiply both sides by $5y - 7$:

$$x(5y - 7) = 3y + 2$$

Distribute x:

$$5xy - 7x = 3y + 2$$

Rearrange to get all terms involving y on one side:

$$5xy - 3y = 7x + 2$$

Factor out y from the left-hand side:

$$y(5x - 3) = 7x + 2$$

Solve for y:

$$y = \frac{7x + 2}{5x - 3}$$

4. Replace y with $f^{-1}(x)$:

$$f^{-1}(x) = \frac{7x + 2}{5x - 3}$$

Thus, the inverse of $f(x) = \frac{3x+2}{5x-7}$ is:

$$f^{-1}(x) = \frac{7x + 2}{5x - 3}$$

Problem 5

Evaluate $y = \tan[\sin^{-1}\left(-\frac{1}{3}\right)]$

To evaluate $y = \tan[\sin^{-1}\left(-\frac{1}{3}\right)]$, we proceed as follows:

1. Determine the angle θ for $\sin^{-1}\left(-\frac{1}{3}\right)$:

$$\theta = \sin^{-1}\left(-\frac{1}{3}\right)$$

This means $\sin(\theta) = -\frac{1}{3}$.

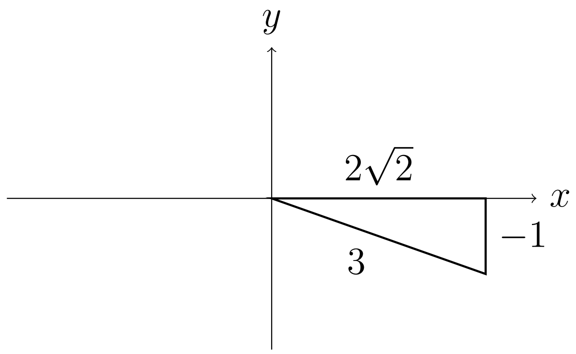
2. Construct a right triangle in Q3.

$$y = -1$$

$$r = 3$$

$$x = ?$$

$$x = \sqrt{3^2 - (-1)^2} = \sqrt{9-1} = \sqrt{8} = 2\sqrt{2}$$



3. Calculate $\tan(\theta)$:

$$\tan(\theta) = \frac{\text{y}}{\text{x}} = \frac{-1}{2\sqrt{2}} = \frac{-1}{2\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{-\sqrt{2}}{4}$$

Therefore,

$$y = \tan[\sin^{-1}\left(-\frac{1}{3}\right)] = \frac{-\sqrt{2}}{4}.$$

Problem 6

Graph $f(x) = \dfrac{x^2}{(x+4)(x-1)}$

Domain: $(-\infty, -4) \cup (-4, 1) \cup (1, \infty)$

VA: $x = -4$ and $x = 1$

HA: $y = 1$

x-intercept: $(0, 0)$

y-intercept: $(0, 0)$

Long Division:

$$
\begin{array}{r|l}
 & 1 \\
\hline
x^2+3x-4) & x^2 \\
 & -x^2-3x+4 \\
\hline
 & -3x+4
\end{array}
$$

Long Division yields: $f(x) = 1 + \frac{-3x+4}{(x+4)(x-1)}$. Complete a number line test around the zeros of the numerator and denominator of the remainder function. In this case, the remainder function is $\frac{-3x+4}{(x+4)(x-1)}$. On the number line, test around -4, 1 and $\frac{4}{3}$.

NLT: + - + -

At $x = \frac{4}{3}$, the graph crosses the HA.