Trig Identities

$$sin^2(x) + cos^2(x) = 1$$
$$1 + tan^2(x) = sec^2(x)$$
$$1 + cot^2(x) = csc^2(x)$$
$$sin^2(x) = \frac{1}{2}(1 - \cos(2x))$$
$$cos^2(x) = \frac{1}{2}(1 + \cos(2x))$$
$$\cos(2x) = cos^2(x) - sin^2(x)$$
$$\sin(2x) = 2\sin(x)\cos(x)$$

Linearization

$$L(x) = f(a) + f'(x)(x - a)$$

Derivatives

$$\frac{dy}{dx}\cot(x) = -csc^2(x)$$
$$\frac{dy}{dx}\sec(x) = \sec(x)\tan(x)$$
$$\frac{dy}{dx}\csc(x) = -\csc(x)\cot(x)$$
$$\frac{dy}{dx}\tan(x) = sec^2(x)$$
$$\frac{dy}{dx}\arcsin(x) = \frac{1}{\sqrt{1 - x^2}}$$
$$\frac{dy}{dx}\text{arcsec}(x) = \frac{1}{|x|\sqrt{x^2 - 1}}$$
$$\frac{dy}{dx}\arctan(x) = \frac{1}{1 + x^2}$$

Product Rule

$$\frac{d}{dx}(uv) = u\frac{dv}{dx} + v\frac{du}{dx}$$

Quotient Rule

$$\frac{d}{dx}\left(\frac{u}{v}\right) = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Integrals

$$\int b^{ax}dx = \frac{b^{ax}}{aln(b)} + C$$
$$\int \ln(x)\,dx = xln(x) - x + C$$
$$\int \tan(x)\,dx = -\ln|\cos(x)| + C$$
$$\int \cot(x)\,dx = \ln|\sin(x)| + C$$
$$\int \csc(x)\,dx = -\ln|\csc(x) + \cot(x)| + C$$
$$\int \sec(x)\,dx = \ln|\sec(x) + \tan(x)| + C$$

Linear Differential Equations

$$\frac{dy}{dx} + P(x)y = Q(x)$$
$$v(x) = e^{\int P(x)dx}$$
$$(v(x)y)' = v(x)Q(x)$$

Riemann Sums

$$x_k = x_0 + k\Delta x$$
$$RH: \sum_{k=1}^{n} \Delta x f(x_k)$$
$$LH: \sum_{k=1}^{n-1} \Delta x f(x_k)$$
$$MP: \sum_{k=0}^{n-1} \Delta x\, f\left(\frac{x_k + x_{k+1}}{2}\right)$$
$$TZ: 2\sum_{k=1}^{n-1}[f(x_k) + f(x_n)] + f(x_0)\Delta x$$
$$SP: \frac{\Delta x}{3}[f(x_0 + 4f(x_1) + 2f(x_2) + \cdots + f(x_{n)}$$

Riemann Sums error

$$RH: \frac{1}{2}\frac{(b-a)^2}{n}f'(c)$$
$$LH: \frac{1}{2}\frac{(b-a)^2}{n}f'(c)$$
$$MP: \frac{1}{24}\frac{(b-a)^3}{n^2}f''(c)$$
$$TZ: -\frac{1}{12}\frac{(b-a)^3}{n^2}f''(c)$$
$$SP: -\frac{1}{2880}\frac{(b-a)^5}{n^4}f^{(4)}(c)$$

L'Hopital's Rule

$$\lim_{x\to\infty}\frac{f(x)}{g(x)} = \frac{0}{0}\,or\,\frac{\infty}{\infty} => \lim_{x\to\infty}\frac{f(x)}{g(x)} = \lim_{x\to\infty}\frac{f(x)'}{g(x)'}$$

Nth term test

$$\lim_{n\to\infty} \neq 0 \;\to diverges$$

P-series

$$\lim_{p\to\infty}\frac{1}{n^p}, p > 1\; converges$$

Geometric series

$$\sum_{n=0}^{\infty} r^n = \frac{1}{1-r}, |r| < 1 \to converges\ absolutely$$

Ratio test

$$\left|\frac{a_{n+1}}{a_n}\right| = L, L < 1\ converges, L > 1\ diverges$$

Root test

$$\lim_{x\to\infty}|a_n|^{\frac{1}{n}} = L, L < 1\ converges, L > 1\ diverges$$

Direct comparison test

$$f(x) \leq g(x)$$

$$if \int_a^\infty f(x)dx \ diverges \ \rightarrow \int_a^\infty g(x)dx \ diverges$$

$$if \int_a^\infty g(x)dx \ converges$$

$$\rightarrow \int_a^\infty f(x)dx \ converges$$

Limit comparison test

$$f(x) \leq g(x)$$

$$\lim_{n\to\infty} \frac{f(x)}{g(x)} = L$$

$$0 < L < \infty \ \rightarrow both \ converge \ or \ diverge$$

$$L = 0, both \ converge \ || \ L = \infty, both \ diverge$$

Alternating series

$$\sum_{n=0}^\infty a_n, a_n \approx (-1)^n \ldots$$

$$if \ |a_n| \ converges, series \ converges \ absolutely$$

$$if \sum_{n=0}^\infty a_n \rightarrow 0, no - series \ diverges \ absolutely$$

$$If \sum_{n=0}^\infty a_n \ is \ decreasing, converges \ conditionally$$

Taylor series

$$f(x) = \sum_{n=0}^\infty \frac{f^{(n)}(a)}{n!}(x-a)^n$$

Maclaurin series

$$f(x) = \sum_{n=0}^\infty \frac{f^{(n)}(0)}{n!}(x)^n$$

Error formula

$$|R_n(x)| \leq \frac{M(x-a)^{n+1}}{(n+1)!}$$

$$M \geq |f^n(x)|$$

Important Taylor series

$$\frac{1}{1-x} = \sum_{n=0}^\infty x^n$$

$$e^x = \sum_{n=0}^\infty \frac{x^n}{n!}$$

$$\sin(x) = \sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!}$$

$$\cos(x) = \sum_{n=0}^\infty (-1)^n \frac{x^{2n}}{(2n)!}$$

$$\arctan(x) = \sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{2n+1}$$

$$\ln(1-x) = \sum_{n=1}^\infty -\frac{x^n}{n}$$

Area

$$\int_a^b (top) - (bottom)dx$$

Volumes (Disk/Washer)

$$\int_a^b \pi[R(x)]^2 dx = \int_a^b \pi([R(x)]^2 - [r(x)]^2)dx$$

Volumes (Shell)

$$\int_a^b 2\pi(rh)dx$$