In-Class Final Exam Review Set A, Math 1554, Fall 2019

1. Indicate whether the statements are true or false.

true	false	
○	○	If a linear system has more unknowns than equations, then the system has either no solutions or infinitely many solutions.
○	○	A $n \times n$ matrix A and its echelon form E will always have the same eigenvalues.
○	○	$x^2 - 2xy + 4y^2 \geq 0$ for all real values of x and y.
○	○	If matrix A has linearly dependent columns, then $\dim((\text{Row}A)^\perp) > 0$.
○	○	If λ is an eigenvalue of A, then $\dim(\text{Null}(A - \lambda I)) > 0$.
○	○	If A has QR decomposition $A = QR$, then $\text{Col}A = \text{Col}Q$.
○	○	If A has LU decomposition $A = LU$, then $\text{rank}(A) = \text{rank}(U)$.
○	○	If A has LU decomposition $A = LU$, then $\dim(\text{Null}\,A) = \dim(\text{Null}\,U)$).

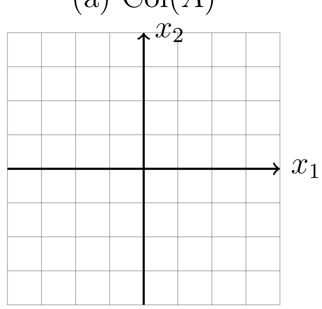
2. Give an example of the following.

   i) A 4×3 lower triangular matrix, A. such that $\text{Col}(A)^\perp$ is spanned by the vector $\vec{v} = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 1 \end{pmatrix}$. $\quad A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

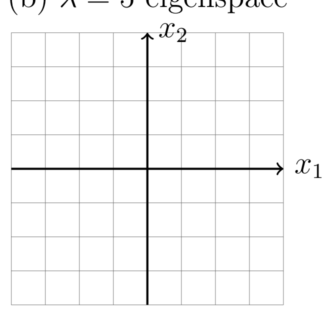
   ii) A 3×4 matrix A, that is in RREF, and satisfies $\dim\left((\text{Row}\,A)^\perp\right) = 2$ and $\dim\left((\text{Col}\,A)^\perp\right) = 2$. $A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$

3. (3 points) Suppose $A = \begin{pmatrix} 3 & 1 \\ 6 & 2 \end{pmatrix}$. On the grid below, sketch a) $\text{Col}(A)$, and b) the eigenspace corresponding to eigenvalue $\lambda = 5$.

(a) $\text{Col}(A)$



(b) $\lambda = 5$ eigenspace



4. Fill in the blanks.

   (a) If $A \in \mathbb{R}^{M \times N}$, $M < N$, and $A\vec{x} = 0$ does not have a non-trivial solution, how many pivot columns does A have? $\boxed{}$

   (b) Consider the following linear transformation.

$$T(x_1, x_2) = (2x_1 - x_2,\ 4x_1 - 2x_2,\ x_2 - 2x_1)\,.$$

   The domain of T is $\boxed{}$. The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \end{pmatrix}$. The co-domain of T is $\boxed{}$. The range of T is:

5. Four points in $\mathbb{R}^2$ with coordinates (t, y) are $(0, 1)$, $(\frac{1}{4}, \frac{1}{2})$, $(\frac{1}{2}, -\frac{1}{2})$, and $(\frac{3}{4}, -\frac{1}{2})$. Determine the values of c_1 and c_2 for the curve $y = c_1 \cos(2\pi t) + c_2 \sin(2\pi t)$ that best fits the points. Write the values you obtain for c_1 and c_2 in the boxes below.

$$c_1 = \boxed{} \qquad c_2 = \boxed{}$$