In-Class Final Exam Review Set B, Math 1554, Fall 2019

1. Indicate whether the statements are true or false.

true	false	
○	○	For any vector $\vec{y} \in \mathbb{R}^2$ and subspace W, the vector $\vec{v} = \vec{y} - \text{proj}_W \vec{y}$ is orthogonal to W.
○	○	If A is $m \times n$ and has linearly dependent columns, then the columns of A cannot span $\mathbb{R}^m$.
○	○	If a matrix is invertible it is also diagonalizable.
○	○	If E is an echelon form of A, then $\text{Null}\,A = \text{Null}\,E$.
○	○	If the SVD of $n \times n$ singular matrix A is $A = U\Sigma V^T$, then $\text{Col}A = \text{Col}U$.
○	○	If the SVD of $n \times n$ matrix A is $A = U\Sigma V^T$, $r = \text{rank}A$, then the first r columns of V give a basis for $\text{Null}A$.

2. Give an example of:

   a) a vector $\vec{u} \in \mathbb{R}^3$ such that $\text{proj}_{\vec{p}}\vec{u} = \vec{p}$, where $\vec{u} \neq \vec{p}$, and $\vec{p} = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$: $\quad \vec{u} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$

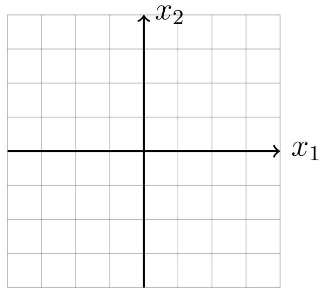
   b) an upper triangular 4×4 matrix A that is in RREF, 0 is its only eigenvalue, and its corresponding eigenspace is 1-dimensional. $\quad A = \begin{pmatrix} & & & \\ & & & \\ & & & \\ & & & \end{pmatrix}$

   c) A 3×4 matrix, A, and $\text{Col}(A)^\perp$ is spanned by $\begin{pmatrix} 1 \\ -3 \\ -4 \end{pmatrix}$.

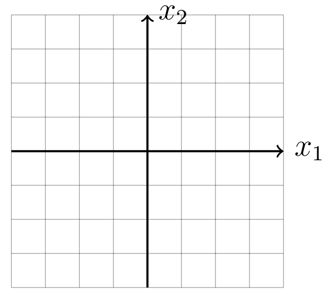
   d) A 2×2 matrix in RREF that is diagonalizable and not invertible.

3. Suppose $A = \begin{pmatrix} 2 & -1 \\ 4 & -2 \end{pmatrix}$. On the grid below, sketch a) the range of $x \to Ax$, b) $(\text{Col}\, A)^{\perp}$, (c) set of solutions to $A\vec{x} = \begin{pmatrix} 3 \\ 6 \end{pmatrix}$.

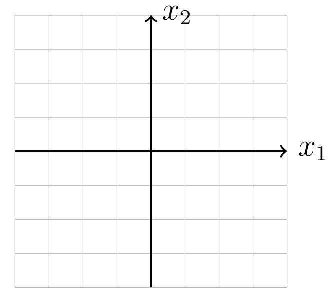
(a) range



(b) $(\text{Col}\, A)^{\perp}$



(c) solutions



4. Matrix A is a 2×2 matrix whose eigenvalues are $\lambda_1 = \frac{1}{2}$ and $\lambda_2 = 1$, and whose corresponding eigenvectors are $\vec{v}_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \vec{v}_2 = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$. Calculate

   1. $A(\vec{v}_1 + 4\vec{v}_2)$
   2. A^{10}
   3. $\lim_{k\to\infty} A^k(\vec{v}_1 + 4\vec{v}_2)$