

## Learning Objectives

Learning objectives articulate what students are **expected to be able to do** in a course. This course has **course-level** learning objectives that are stated in the syllabus, and **section-level** learning objectives that are stated below.

### Course-Level Learning Objectives

Course-level learning objectives were stated in the syllabus. Throughout this course, its expected that students will be able to do the following.

- A) Construct, or give examples of, mathematical expressions that involve vectors, matrices, and linear systems of linear equations.
- B) Evaluate mathematical expressions to compute quantities that deal with linear systems and eigenvalue problems.
- C) Analyze mathematical statements and expressions (for example, to assess whether a particular statement is accurate, or to describe solutions of systems in terms of existence and uniqueness).
- D) Write logical progressions of precise mathematical statements to justify and communicate your reasoning.
- E) Apply linear algebra concepts to model, solve, and analyze real-world situations.

### Section-Level Learning Objectives

Section-level learning objectives pertain to specific sections of our textbook. They state what students are expected to be able to do specifically for the topics in each section.

## 1 Linear Equations

### 1.1 Systems of Linear Equations

- 1. Apply elementary row operations to solve linear systems of equations.
- 2. Characterize a linear system in terms of the number of solutions, and whether the system is consistent or inconsistent.
- 3. Express a set of linear equations as an augmented matrix.

### 1.2 Row Reduction and Echelon Forms

- 1. Characterize a linear system in terms of the number of leading entries, free variables, pivots, pivot columns, pivot positions.
- 2. Apply the row reduction algorithm to reduce a linear system to echelon form, or reduced echelon form.
- 3. Apply the row reduction algorithm to compute the coefficients of a polynomial.

### 1.3 Vector Equations

1. Apply geometric and algebraic properties of vectors in  $\mathbb{R}^n$  to compute vector additions and scalar multiplications.
2. Characterize a set of vectors in terms of **linear combinations**, their **span**, and how they are related to each other geometrically.
3. Characterize the solutions to a linear system in terms of linear combinations and span.

### 1.4 The Matrix Equation

1. Compute matrix-vector products.
2. Express linear systems as vector equations and matrix equations.
3. Characterize linear systems and sets of vectors using the concepts of span, linear combinations, and pivots.

### 1.5 Solution Sets of Linear Systems

1. Express the solution set of a linear system in parametric vector form.
2. Provide a geometric interpretation to the solution set of a linear system.
3. Characterize homogeneous linear systems using the concepts of free variables, span, pivots, linear combinations, and echelon forms.

### 1.7 Linear Independence

1. Characterize a set of vectors and linear systems using the concept of linear independence.
2. Construct dependence relations between linearly dependent vectors.

### 1.8 An Introduction to Linear Transforms

1. Construct and interpret linear transformations in  $\mathbb{R}^2$  or  $\mathbb{R}^3$  (for example, interpret a linear transform as a projection, or as a shear).
2. Characterize linear transforms using the concepts of existence and uniqueness.

### 1.9 Linear Transforms

1. Identify and construct linear transformations of a matrix.
2. Characterize linear transformations as onto, one-to-one.
3. Solve linear systems represented as linear transforms.
4. Express linear transforms in other forms, such as as matrix equations, and vector equations.

## 2 Matrix Algebra

### 2.1 Matrix Operations

1. Apply matrix algebra, the matrix transpose, and the zero and identity matrices, to solve and analyze matrix equations

### 2.2 Inverse of a Matrix

1. Apply the formal definition of an inverse, and its algebraic properties, to solve and analyze linear systems.
2. Compute the inverse of an  $n \times n$  matrix, and use it to solve linear systems.

### 2.3 Invertible Matrices

1. Characterize the invertibility of a matrix using the Invertible Matrix Theorem.

### 2.5 Matrix Factorizations

1. Compute an  $LU$  factorization of a matrix.
2. Apply the  $LU$  factorization to solve systems of equations.

### 2.8 Subspaces of $\mathbb{R}^n$

1. Determine whether a set is a subspace.
2. Determine whether a vector is in a particular subspace, or find a vector in that subspace.
3. Construct a basis for a subspace (for example, a basis for  $\text{Col}(A)$ ).

### 2.9 Dimension and Rank

1. Calculate the coordinates of a vector in a given basis.
2. Characterize a subspace using the concept of dimension.
3. Characterize a matrix using the concepts of rank, column space, null space.
4. Apply the Rank, Basis, and Matrix Invertibility theorems to describe matrices and subspaces.

## 3 Determinants

### 3.1 Introduction to Determinants

1. Compute determinants of  $n \times n$  matrices using a cofactor expansion.
2. Apply theorems to compute determinants of matrices that have particular structures.

### **3.2 Properties of the Determinant**

1. Apply properties of determinants (related to row reductions, transpose, and matrix products) to compute determinants.
2. Use determinants to determine whether a square matrix is invertible.