Midterm 1, 1:30, Math 1554, Spring 2020

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

GTID Number: ______________________

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Section Number (e.g. A4, M2, QH3, etc.) _______ TA Name _________________________

Circle your instructor:

Dr. Belegradek, Dr. Mayer, Dr. Barone

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.

You do not need to justify your reasoning for questions on this page.

1. (7 points) Suppose A, B are matrices and b, u, v are vectors such that their products in the questions below are defined, and that matrix A is $m \times n$. Select true if the statement is **true** for all A, B, b, u, v. Otherwise, select **false**.

		true	false
i)	If columns of A span $\mathbb{R}^m$, then $Ax = b$ is consistent for every b.	○	○
ii)	If A is a 6×4 matrix with 4 pivotal rows, then $Ax = 0$ has infinitely many solutions.	○	○
iii)	If A has fewer rows than columns, then $Ax = b$ has infinitely many solutions.	○	○
iv)	If $Ax = b$ is consistent, then so is $Ax = -b$.	○	○
v)	If u, v are linearly dependent vectors, then Au, Av are also linearly dependent.	○	○
vi)	If u, v are linearly independent vectors, then so are $u + v, u$.	○	○
vii)	If A, B are matrices and $AB = 0$, then either $A = 0$ or $B = 0$.	○	○

2. (3 points) Indicate whether the following situations are possible or impossible.

		possible	impossible
i)	A is a 5×3 matrix with linearly independent columns.	○	○
ii)	A and B are 2×2 matrices, but $AB \neq BA$.	○	○
iii)	Matrix A has a pivot in the last column and the system $A\vec{x} = \vec{b}$ is consistent.	○	○

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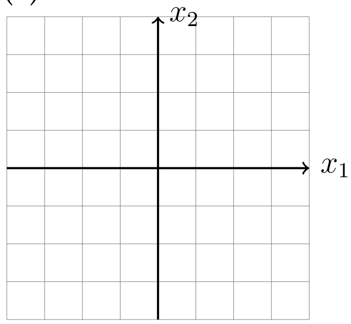
3. (2 points) Suppose $A = \begin{pmatrix} 1 & 3 \\ 1 & 3 \end{pmatrix}$. On the grid below, sketch

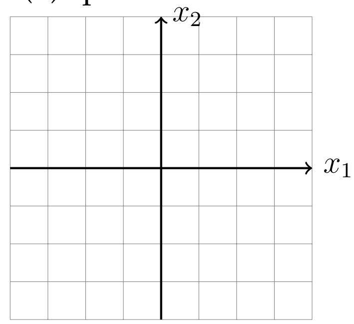
a) any non-zero vector that is a solution to $A\vec{x} = \vec{0}$,

b) the span of the columns of A.

(a) Non-Zero Solution



(b) span of columns



4. (6 points) If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible*. You do not need to justify your reasoning.

(a) A 3×3 matrix A in RREF whose only pivots are in the second and third columns.

$$A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

(b) A 3×2 matrix A in RREF such that $Ax = b$ is consistent for every b in $\mathbb{R}^3$.

$$A = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$$

(c) The 2×2 matrix A such that the linear transformation $T(x) = Ax$ first rotates counterclockwise by $\frac{\pi}{2}$ radians and then reflects through the line $x_2 = 0$.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

You do not need to justify your reasoning for questions on this page.

5. (12 points) Fill in the blanks.

   (a) If A is 5×7 and has exactly 4 pivots, how many free variables does $A\vec{x} = \vec{0}$ have?

   [ ]

   (b) If A is an $m \times n$ matrix with $m < n$, and $A\vec{x} = \vec{b}$ has a solution for all $\vec{b}$, how many pivot columns does A have?

   [ ]

   (c) Consider the following linear transformation.

   $$T(x_1, x_2) = (3x_1 - 2x_2,\ 3x_2,\ 3x_1 - 6x_2)\,.$$

   - The domain of T is [ ].
   - The codomain of T is [ ].
   - The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \\ \\ \\ \end{pmatrix}$.
   - The standard matrix A associated to T is

   $$A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}.$$

   - A particular solution to $T(\vec{x}) = \begin{pmatrix} 4 \\ 3 \\ 0 \end{pmatrix}$ is $\vec{x} = \begin{pmatrix} \\ \\ \end{pmatrix}$.
   - Is T onto (yes or no)? [ ]
   - Is T one to one (yes or no)? [ ]

   (d) Suppose A, B, and C are matrices. A is size 5×3, C is size 5×7, and $AB = C$.

   - How many rows does B have? [ ]
   - How many columns does B have? [ ]

   (e) List all possible values of k such that $AB = BA$.

   $$A = \begin{pmatrix} 3 & 1 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ k & 3 \end{pmatrix}, \qquad k = \boxed{}$$

6. (5 points) Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 1 & -1 & 0 & -4 & 2 \\ 0 & 0 & 0 & 2 & 2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$$

(a) Row reduce the augmented matrix $(\, A \,|\, \vec{b} \,)$ to RREF.

(b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation. Show your work.

7. (5 points) For what value(s) of t is $\vec{b}$ in the plane spanned by $\vec{a}_1$ and $\vec{a}_2$? Show your work.

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ t \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 0 \\ 1 \\ -2t \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} t \\ 1 \\ 3 \end{pmatrix}; \qquad t = \boxed{}$$