Midterm 1, 3:00, Math 1554, Spring 2020

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

GTID Number: ______________________

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Section Number (e.g. A4, M2, QH3, etc.) ________ TA Name __________________________

Circle your instructor:

Dr. Belegradek, Dr. Mayer, Dr. Barone

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.

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You do not need to justify your reasoning for questions on this page.

1. (7 points) Suppose A, B are matrices and b, u, v are vectors such that their products in the questions below are defined, and that matrix A is $m \times n$. Select true if the statement is **true** for all A, B, b, u, v. Otherwise, select **false**.

		true	false
i)	If $Ax = 0$ has a nonzero solution, then it has infinitely many solutions.	○	○
ii)	If A is a 5×6 matrix with 4 pivotal columns, then $Ax = b$ is not consistent for some b.	○	○
iii)	If A has fewer rows than columns, then $Ax = 0$ has infinitely many solutions.	○	○
iv)	If $Ax = 2b$ is consistent, then so is $Ax = b$.	○	○
v)	If T is a linear transformation, then $T(0) = 0$.	○	○
vi)	If u, v, b are linearly independent vectors, then so are u, v.	○	○
vii)	If A is a matrix such that $AA^T = A^TA$, then A is a square matrix.	○	○

2. (3 points) Indicate whether the following situations are possible or impossible.

		possible	impossible
i)	A is a 3×4 matrix with linearly dependent columns.	○	○
ii)	A and B are 2×2 matrices, and $AB \neq BA$.	○	○
iii)	$n \times n$ matrix A has a pivot in every row and the system $A\vec{x} = \vec{b}$ is inconsistent, where $\vec{x}$ and $\vec{b}$ are vectors in $\mathbb{R}^n$.	○	○

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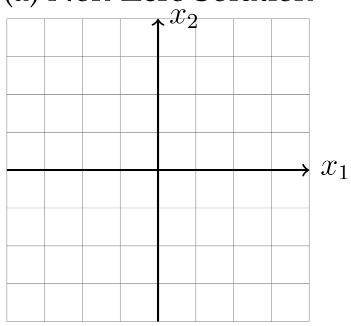
3. (2 points) Suppose $A = \begin{pmatrix} 2 & 3 \\ 4 & 6 \end{pmatrix}$. On the grid below, sketch

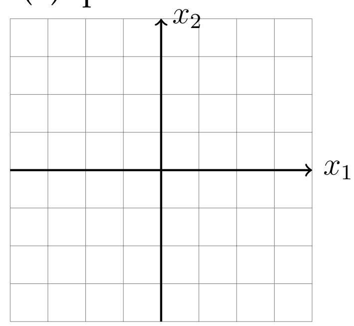
   a) any non-zero vector that is a solution to $A\vec{x} = \vec{0}$,

   b) the span of the columns of A.

(a) Non-Zero Solution



(b) span of columns



4. (6 points) If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write down *not possible*. You do not need to justify your reasoning.

   (a) A 3×3 matrix A in RREF such that $Ax = 0$ has exactly one free variable.

   $$A = \begin{pmatrix} \quad & \quad & \quad \\ & & \\ & & \end{pmatrix}$$

   (b) A 3×2 matrix A in RREF such that $Ax = b$ is consistent for every b in $\mathbb{R}^3$.

   $$A = \begin{pmatrix} \quad & \quad \\ & \\ & \end{pmatrix}$$

   (c) The 2×2 matrix A such that the linear transformation $T(x) = Ax$ first projects onto the x_1 axis, and then rotates counterclockwise by $\frac{\pi}{2}$.

   $$A = \begin{pmatrix} \quad & \quad \\ & \end{pmatrix}$$

You do not need to justify your reasoning for questions on this page.

5. (12 points) Fill in the blanks.

(a) If A is 5×4 and has exactly 2 pivots, how many free variables does $A\vec{x} = \vec{0}$ have?

[ ]

(b) If A is an $m \times n$ matrix with $m < n$, and $A\vec{x} = \vec{b}$ has a solution for all $\vec{b}$, how many pivot columns does A have?

[ ]

(c) Consider the following linear transformation.

$$T(x_1, x_2, x_3) = (x_1 + 4x_2 - x_3,\ 2x_1 + 8x_2 - 2x_3)\,.$$

- The domain of T is [ ].
- The codomain of T is [ ].
- The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \quad \end{pmatrix}$.
- A particular solution to $T(\vec{x}) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ is $\vec{x} = \begin{pmatrix} \\ \\ \\ \end{pmatrix}$.
- The standard matrix A associated to T is

$$A = \begin{pmatrix} \quad & \quad & \quad \\ \quad & \quad & \quad \end{pmatrix}.$$

- Is T onto (yes or no)? [ ].
- Is T one to one (yes or no)? [ ].

(d) Suppose A, B, and C are matrices. A is size 2×6, C is size 2×3, and $AB = C$.

- How many rows does B have? [ ]
- How many columns does B have? [ ]

(e) List all possible values of k such that $AB = BA$.

$$A = \begin{pmatrix} 4 & 2 \\ 0 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ k & 2 \end{pmatrix}, \qquad k = \boxed{\quad}$$

6. (5 points) Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 1 & 0 & -4 & 2 \\ 0 & 0 & 4 & 8 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 8 \\ 4 \end{pmatrix}$$

(a) Row reduce the augmented matrix $(\, A \,|\, \vec{b} \,)$ to RREF.

(b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation. Show your work.

7. (5 points) For what value(s) of k are $\vec{a}_1$, $\vec{a}_2$, and $\vec{a}_3$ linearly dependent? Show your work.

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ k \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 0 \\ 1 \\ -k \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} -k \\ 1 \\ -2 \end{pmatrix}; \qquad k = \boxed{}$$