

Midterm 1, 4:30, Math 1554, Spring 2020

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name _____ Last Name _____

GTID Number: _____

Student GT Email Address: _____@gatech.edu

Section Number (e.g. A4, M2, QH3, etc.) _____ TA Name _____

Circle your instructor:

Dr. Belegradek, Dr. Mayer, Dr. Barone

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.

Math 1554, Midterm 1, 4:30. Your initials: _____

You do not need to justify your reasoning for questions on this page.

1. (7 points) Suppose A, B, C are matrices and b, u, v are vectors such that their products in the questions below are defined, and that matrix A is $m \times n$. Select true if the statement is **true** for all A, B, b, u, v . Otherwise, select **false**.

	true	false
i) If $Ax = 0$ has a unique solution, then the columns of A are linearly independent.	<input type="radio"/>	<input type="radio"/>
ii) If the system $Ax = b$ has a nonzero solution, then it has infinitely many solutions.	<input type="radio"/>	<input type="radio"/>
iii) If $Ax = b$ is consistent for every b , then the columns of A span \mathbb{R}^m .	<input type="radio"/>	<input type="radio"/>
iv) If $Ax = b$ is consistent, then so is $Ax = 3b$.	<input type="radio"/>	<input type="radio"/>
v) If A, B, C are matrices and $AB = AC$, then $B = C$.	<input type="radio"/>	<input type="radio"/>
vi) If u, v are linearly independent vectors, then Au, Av are also linearly independent.	<input type="radio"/>	<input type="radio"/>
vii) If u, v, b are linearly dependent vectors, then so are u, v .	<input type="radio"/>	<input type="radio"/>

2. (3 points) Indicate whether the following situations are possible or impossible.

	possible	impossible
i) A is a 4×5 matrix with linearly independent columns.	<input type="radio"/>	<input type="radio"/>
ii) A and B are 2×2 matrices and $AB = BA$.	<input type="radio"/>	<input type="radio"/>
iii) The augmented matrix $[A b]$ has a pivot in the last column and the system $A\vec{x} = \vec{b}$ is inconsistent.	<input type="radio"/>	<input type="radio"/>

Math 1554, Midterm 1, 4:30. Your initials: _____

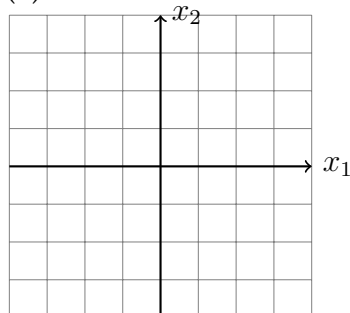
You do not need to justify your reasoning for questions on this page.

3. (2 points) Suppose $A = \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix}$. On the grid below, sketch

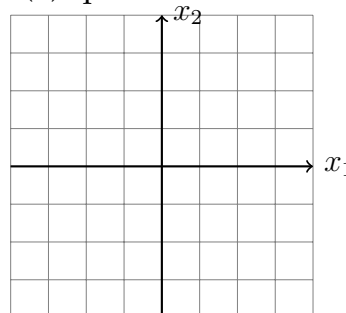
a) any non-zero vector that is a solution to $A\vec{x} = \vec{0}$,

b) the span of the columns of A .

(a) Non-Zero Solution



(b) span of columns



4. (6 points) If possible, write down an example of a matrix with the following properties. If it is not possible to do so, write *not possible*. You do not need to justify your reasoning.

(a) A 3×3 matrix A in RREF whose only pivot is in the third column.

$$A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

(b) A 3×2 matrix A in RREF such that $Ax = 0$ has a unique solution.

$$A = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$$

(c) The 2×2 matrix A such that the linear transformation $T(x) = Ax$ first rotates counter-clockwise by $\frac{\pi}{2}$ and then projects onto the x_2 axis.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

Math 1554, Midterm 1, 4:30. Your initials: _____

You do not need to justify your reasoning for questions on this page.

5. (12 points) Fill in the blanks.

(a) If A is 6×5 and has exactly 2 pivots, how many free variables does $A\vec{x} = \vec{0}$ have?

(b) If A is an $m \times n$ matrix with $m < n$, and $A\vec{x} = \vec{b}$ has a solution for all \vec{b} , how many pivot columns does A have?

(c) Consider the following linear transformation.

$$T(x_1, x_2) = (3x_1 + x_2, 2x_1, x_1 - x_2).$$

• The domain of T is .

• The codomain of T is .

• The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \\ \\ \end{pmatrix}$.

• A particular solution to $T(\vec{x}) = \begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ is $\vec{x} = \begin{pmatrix} \\ \end{pmatrix}$.

• The standard matrix A associated to T is

$$A = \begin{pmatrix} & \\ & \\ & \end{pmatrix}.$$

• Is T onto (yes or no)? .

• Is T one to one (yes or no)? .

(d) Suppose A , B , and C are matrices. A is size 3×5 , C is size 3×2 , and $AB = C$.

• How many rows does B have?

• How many columns does B have?

(e) List all possible values of k such that $AB = BA$.

$$A = \begin{pmatrix} 3 & 1 \\ 1 & 0 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 0 \\ k & 3 \end{pmatrix}, \quad k = \text{$$

Math 1554, Midterm 1, 4:30. Your initials: _____

6. (5 points) Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 2 & -4 & -4 & 0 \\ 0 & 2 & 8 & 0 \end{pmatrix}, \vec{b} = \begin{pmatrix} 6 \\ 6 \end{pmatrix}$$

(a) Express the augmented matrix $(A|\vec{b})$ in RREF.

(b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation. Show your work.

Math 1554, Midterm 1, 4:30. Your initials: _____

7. (5 points) For what value(s) of p is \vec{a}_3 , a linear combination of vectors \vec{a}_1 , and \vec{a}_2 ? Show your work.

$$\vec{a}_1 = \begin{pmatrix} 1 \\ 0 \\ 2p \end{pmatrix}, \quad \vec{a}_2 = \begin{pmatrix} 0 \\ 1 \\ p \end{pmatrix}, \quad \vec{a}_3 = \begin{pmatrix} -p \\ 2 \\ -4 \end{pmatrix}; \quad p = \boxed{}$$