Midterm 1, 6:00, Math 1554, Spring 2020

PLEASE PRINT YOUR NAME CLEARLY IN ALL CAPITAL LETTERS

First Name ____________________ Last Name ____________________

GTID Number: ______________________

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Section Number (e.g. A4, M2, QH3, etc.) ________ TA Name __________________________

Circle your instructor:

Dr. Belegradek, Dr. Mayer, Dr. Barone

Student Instructions

- **Show your work** and justify your answers for all questions unless stated otherwise.
- **Organize your work** in a reasonably neat and coherent way.
- Electronic devices are not allowed.
- Use dark and clear writing: your exam will be scanned into a digital system.
- Exam pages are double sided. Be sure to complete both sides.
- Leave a 1 inch border around the edges of exams.

Math 1554, Midterm 1, 6:00. Your initials: ______________

You do not need to justify your reasoning for questions on this page.

1. (7 points) Suppose A, B are matrices and b, u, v are vectors such that their products in the questions below are defined, and that matrix A is $m \times n$. Select true if the statement is **true** for all A, B, b, u, v. Otherwise, select **false**.

		true	false
i)	If $Ax = b$ has a unique solution, then A has independent columns.	◯	◯
ii)	If $Ax = b$ has at least two solutions, then $Ax = b$ has infinitely many solutions.	◯	◯
iii)	If $Ax = 0$ has a unique solution, then so does $Ax = b$.	◯	◯
iv)	If $Ax = -b$ is consistent, then so is $Ax = b$.	◯	◯
v)	If $Au = 0 = Av$, then $2u - 3v$ is a solution of $Ax = 0$.	◯	◯
vi)	If a vector u lies in the span of the vectors v, b, then u, v, b are linearly dependent.	◯	◯
vii)	If u, v are linearly dependent vectors, then Au, Av are also linearly dependent.	◯	◯

2. (3 points) Indicate whether the following situations are possible or impossible.

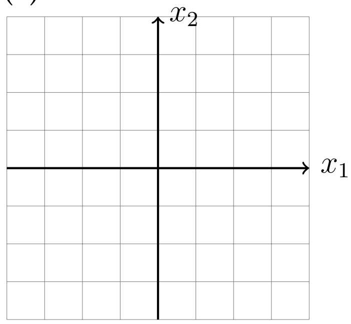
		possible	impossible
i)	A is a 4×5 matrix with linearly dependent columns.	◯	◯
ii)	A and B are 2×2 matrices with $AB = BA$.	◯	◯
iii)	Matrix A has linearly independent columns and $Ax = 0$ has a unique solution.	◯	◯

You do not need to justify your reasoning for questions on this page.

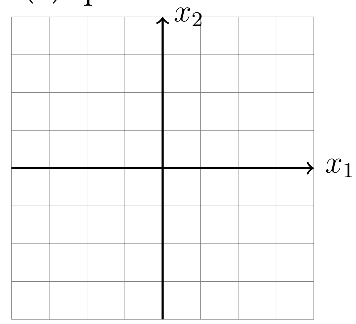
3. (2 points) Suppose $A = \begin{pmatrix} 2 & 4 \\ 1 & 2 \end{pmatrix}$. On the grid below, sketch

   a) any non-zero vector that is a solution to $A\vec{x} = \vec{0}$,
   b) the span of the columns of A.

(a) Non-Zero Solution



(b) span of columns



4. (6 points) If possible, write down an example of a matrix or vector with the following properties. If it is not possible to do so, write *not possible*.

   (a) A 3×3 matrix A in RREF such that $Ax = 0$ has exactly two free variables.

$$A = \begin{pmatrix} & & \\ & & \\ & & \end{pmatrix}$$

   (b) A 3×2 matrix A in RREF such that the linear map $T(x) = Ax$ is onto.

$$A = \begin{pmatrix} & \\ & \\ & \end{pmatrix}$$

   (c) The 2×2 matrix A such that the linear transformation $T(x) = Ax$ first projects onto the x_1 axis, and then reflects about the line $x_2 = x_1$.

$$A = \begin{pmatrix} & \\ & \end{pmatrix}$$

You do not need to justify your reasoning for questions on this page.

5. (12 points) Fill in the blanks.

(a) If A is 7×5 and has exactly 4 pivots, how many free variables does $A\vec{x} = \vec{0}$ have?

[ ]

(b) If A is an $m \times n$ matrix with $m < n$, and $A\vec{x} = \vec{b}$ has a solution for all $\vec{b}$, how many pivot columns does A have?

[ ]

(c) Consider the following linear transformation.

$$T(x_1, x_2, x_3) = (x_1 + x_2 - x_3,\ x_1 + 2x_3)\,.$$

- The domain of T is [ ].
- The codomain of T is [ ].
- The image of $\vec{x} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$ under $T(\vec{x})$ is $\begin{pmatrix} \quad \end{pmatrix}$.
- A particular solution to $T(\vec{x}) = \begin{pmatrix} 2 \\ 3 \end{pmatrix}$ is $\vec{x} = \begin{pmatrix} \quad \end{pmatrix}$.
- The standard matrix A associated to T is

$$A = \begin{pmatrix} \quad \end{pmatrix}.$$

- Is T onto (yes or no)? [ ].
- Is T one to one (yes or no)? [ ].

(d) Suppose A, B, and C are matrices. A is size 3×5, C is size 3×4, and $AB = C$.

- How many rows does B have? [ ]
- How many columns does B have? [ ]

(e) List all possible values of k such that $AB = BA$.

$$A = \begin{pmatrix} 2 & 1 \\ 0 & 3 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & 0 \\ k & 2 \end{pmatrix}, \qquad k = \boxed{}$$

6. (5 points) Consider the linear system $A\vec{x} = \vec{b}$, where

$$A = \begin{pmatrix} 4 & -4 & 0 & 0 & -4 \\ 0 & 1 & 0 & 4 & -2 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 8 \\ 5 \end{pmatrix}$$

(a) Express the augmented matrix $(\, A \,|\, \vec{b} \,)$ in RREF.

(b) Write the set of solutions to $A\vec{x} = \vec{b}$ in parametric vector form. Your answer must be expressed as a vector equation. Show your work.

7. (5 points) For what value(s) of q is $\vec{x}_3$ in the span of $\vec{x}_1$ and $\vec{x}_2$? Show your work.

$$\vec{x}_1 = \begin{pmatrix} 0 \\ 1 \\ 2q \end{pmatrix}, \quad \vec{x}_2 = \begin{pmatrix} 1 \\ 0 \\ q \end{pmatrix}, \quad \vec{x}_3 = \begin{pmatrix} -q \\ -2 \\ 3 \end{pmatrix}; \qquad q = \boxed{}$$