Learning Objectives

Learning goals, or learning objectives, articulate what students are **expected to be able to do** in a course. This course has **course-level** learning objectives that are stated in the syllabus, and **section-level** learning objectives that are stated in the lecture slides.

Course-Level Learning Objectives

Course-level learning objectives should be stated in the syllabus. Throughout this course, it is expected that students will be able to do the following.

A) Construct, or give examples of, mathematical expressions that involve vectors, matrices, and linear systems of linear equations.

B) Evaluate mathematical expressions to compute quantities that deal with linear systems and eigenvalue problems.

C) Analyze mathematical statements and expressions (for example, to assess whether a particular statement is accurate, or to describe solutions of systems in terms of existence and uniqueness).

D) Write logical progressions of precise mathematical statements to justify and communicate your reasoning.

E) Apply linear algebra concepts to model, solve, and analyze real-world situations.

F) Identify course-related information, policies, and procedures that are contained in the syllabus and related course websites.

Section-Level Learning Objectives

Section-level learning objectives were stated in the lecture slides.

1 Linear Equations

1.1 Systems of Linear Equations

1. Characterize a linear system in terms of the number of solutions, and whether the system is consistent or inconsistent.
2. Apply elementary row operations to solve linear systems of equations.
3. Express a set of linear equations as an augmented matrix.

1.2 Row Reduction and Echelon Forms

1. Characterize a linear system in terms of the number of leading entries, free variables, pivots, pivot columns, pivot positions.

2. Apply the row reduction algorithm to reduce a linear system to echelon form, or reduced echelon form.

3. Apply the row reduction algorithm to compute the coefficients of a polynomial.

1.3 Vector Equations

1. Apply geometric and algebraic properties of vectors in $\mathbb{R}^n$ to compute vector additions and scalar multiplications.

2. Characterize a set of vectors in terms of **linear combinations**, their **span**, and how they are related to each other geometrically.

1.4 The Matrix Equation

1. Compute matrix-vector products.

2. Express linear systems as vector equations and matrix equations.

3. Characterize linear systems and sets of vectors using the concepts of span, linear combinations, and pivots.

1.5 Solution Sets of Linear Systems

1. Express the solution set of a linear system in parametric vector form.

2. Provide a geometric interpretation to the solution set of a linear system.

3. Characterize homogeneous linear systems using the concepts of free variables, span, pivots, linear combinations, and echelon forms.

1.7 Linear Independence

1. Characterize a set of vectors and linear systems using the concept of linear independence.

2. Construct dependence relations between linearly dependent vectors.

1.8 An Introduction to Linear Transforms

1. Construct and interpret linear transformations in $\mathbb{R}^n$ (for example, interpret a linear transform as a projection, or as a shear).

2. Characterize linear transforms using the concepts of

   - existence and uniqueness
   - domain, co-domain and range

1.9 Linear Transforms

1. Identify and construct linear transformations of a matrix.
2. Characterize linear transformations as onto and/or one-to-one.
3. Solve linear systems represented as linear transforms.
4. Express linear transforms in other forms, such as as matrix equations or as vector equations.

2 Matrix Algebra

2.1 Matrix Operations

1. **Apply** matrix algebra, the matrix transpose, and the zero and identity matrices, to **solve** and **analyze** matrix equations.

2.2 Inverse of a Matrix

1. Apply the formal definition of an inverse, and its algebraic properties, to solve and analyze linear systems.
2. Compute the inverse of an $n \times n$ matrix, and use it to solve linear systems.
3. Construct elementary matrices.

2.3 Invertible Matrices

1. Characterize the invertibility of a matrix using the Invertible Matrix Theorem.
2. Construct and give examples of matrices that are/are not invertible.

2.4 Partitioned Matrices

1. Apply partitioned matrices to solve problems regarding matrix invertibility and matrix multiplication.

2.5 Matrix Factorizations

1. Compute an LU factorization of a matrix.
2. Apply the LU factorization to solve systems of equations.
3. Determine whether a matrix has an LU factorization.

2.6 The Leontif Input-Output Model

1. Apply matrix algebra and inverses to solve and analyze Leontif Input-Output problems.

2.8 Subspaces of $\mathbb{R}^n$

1. Determine whether a set is a subspace.
2. Determine whether a vector is in a particular subspace, or find a vector in that subspace.
3. Construct a basis for a subspace (for example, a basis for Col(A))

2.9 Dimension and Rank

1. Calculate the coordinates of a vector in a given basis.
2. Characterize a subspace using the concept of dimension (or cardinality).
3. Characterize a matrix using the concepts of rank, column space, null space.
4. Apply the Rank, Basis, and Matrix Invertibility theorems to describe matrices and subspaces.

3 Determinants

3.1 Introduction to Determinants

1. Compute determinants of $n \times n$ matrices using a cofactor expansion.
2. Apply theorems to compute determinants of matrices that have particular structures.

3.2 Properties of the Determinant

1. Apply properties of determinants (related to row reductions, transpose, and matrix products) to compute determinants.
2. Use determinants to determine whether a square matrix is invertible.

3.3 Volume, Linear Transformations

1. Use determinants to compute the area of a parallelogram, or the volume of a parallelepiped, possibly under a given linear transformation.

4 Vector Spaces

4.9 Applications to Markov Chains

1. Construct stochastic matrices and probability vectors.
2. Model and solve real-world problems using Markov chains (e.g. - find a steady-state vector for a Markov chain)
3. Determine whether a stochastic matrix is regular.

5 Eigenvalues and Eigenvectors

5.1 Eigenvectors and Eigenvalues

1. Verify that a given vector is an eigenvector of a matrix.
2. Verify that a scalar is an eigenvalue of a matrix.
3. Construct an eigenspace for a matrix.
4. Apply theorems related to eigenvalues (for example, to characterize the invertibility of a matrix).

5.2 The Characteristic Equation

1. Construct the characteristic polynomial of a matrix and use it to identify eigenvalues and their multiplicities.
2. Characterize the long-term behaviour of dynamical systems using eigenvalue decompositions.

5.3 Diagonalization

1. Determine whether a matrix can be diagonalized, and if possible diagonalize a square matrix.
2. Apply diagonalization to compute matrix powers.

5.5 Complex Eigenvalues

1. Use complex arithmatic to find complex eigenvectors of real matrices with complex eigenvalues.
2. Identify rotation-dilation matrices and sketch the corresponding linear transform, and find the complex eigenvalues.
3. Apply theorems to characterize matrices with complex eigenvalues.

10.2 The Steady-State Vector and Page Rank

1. Determine whether a stochastic matrix is regular.
2. Apply matrix powers and theorems to characterize the long-term behaviour of a Markov chain.
3. Construct a transition matrix, a Markov Chain, and a Google Matrix for a given web, and compute the PageRank of the web.

6 Orthogonality and Least Squares

6.1 Inner Product, Length, and Orthogonality

1. Compute (a) dot product of two vectors, (b) length (or magnitude) of a vector, (c) distance between two points in $\mathbb{R}^n$, and (d) angles between vectors.

2. Apply theorems related to orthogonal complements, and their relationships to Row and Null space, to characterize vectors and linear systems.

6.2 Orthogonal Sets

1. Apply the concepts of orthogonality to

   (a) compute orthogonal projections and distances,
   (b) express a vector as a linear combination of orthogonal vectors,
   (c) characterize bases for subspaces of $\mathbb{R}^n$, and
   (d) construct orthonormal bases.

6.3 Orthogonal Projections

1. Apply concepts of orthogonality and projections to

   (a) compute orthogonal projections and distances,
   (b) express a vector as a linear combination of orthogonal vectors,
   (c) construct vector approximations using projections,
   (d) characterize bases for subspaces of $\mathbb{R}^n$, and
   (e) construct orthonormal bases.

6.4 The Gram-Schmidt Process

1. Apply the iterative Gram Schmidt Process, and the QR decomposition, to construct an orthogonal basis.

2. Compute the QR factorization of a matrix.

Students are not expected to determine whether a matrix has a QR decomposition.

6.5 Least-Squares Problems

1. Compute general solutions, and least squares errors, to least squares problems using the normal equations and the QR decomposition.

6.6 Applications to Linear Models

1. Apply least-squares to construct a linear model from a set of data points

7 Symmetric Matrices and Quadratic Forms

7.1 Diagonalization of Symmetric Matrices

1. Construct an orthogonal diagonalization of a symmetric matrix, $A = PDP^T$.
2. Construct a spectral decomposition of a matrix.

7.2 Quadratic Forms

1. Characterize and classify quadratic forms using eigenvalues and eigenvectors.
2. Express quadratic forms in the form $Q(\vec{x}) = \vec{x}^T A\vec{x}$.
3. Apply the principle axes theorem to express quadratic forms with no cross-product terms.

7.3 Constrained Optimization

1. Apply eigenvalues and eigenvectors to solve optimization problems that are subject to distance and orthogonality constraints.

7.4 The Singular Value Decomposition

1. Compute the SVD for a rectangular matrix.
2. Apply the SVD to
   - estimate the rank and condition number of a matrix,
   - construct a basis for the four fundamental spaces of a matrix, and
   - construct a spectral decomposition of a matrix.