Math 1552
Spring 2019
Quiz 6 4:30pm
February 28, 2019
Time Limit: 15 Minutes

Name (Print): _____

Canvas email: _____

Teaching Assistant/Section: _____

GT ID:

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: _____

This quiz contains 2 pages (including this cover page) and 2 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this quiz.

You are required to show your work on each problem on this quiz. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Please circle or box in your final answer.

Problem	Points	Score
1	15	
2	5	
Total:	20	

1. (15 points) Evaluate the following improper integral:

$$\int_0^1 x\ln(x)\,dx$$

$$\int_b^1 x\ln x\,dx = \frac{x^2\ln x}{2}\Big|_b^1 - \int_b^1 \frac{x}{2}\,dx = \left[\frac{x^2\ln x}{2} - \frac{x^2}{4}\right]_b^1$$

By Parts: $u = \ln x$ $\quad dv = x\,dx$

$du = \frac{1}{x}dx$ $\quad v = \frac{x^2}{2}$

$$= 0 - \frac{1}{4} - \frac{b^2\ln(b)}{2} + \frac{b^2}{4}$$ +6

$$\int_0^1 x\ln x\,dx = \lim_{b\to 0^+}\int_b^1 x\ln x\,dx = \lim_{b\to 0^+}\left(\frac{-1}{4} - \frac{b^2\ln(b)}{2} + \frac{b^2}{4}\right)$$ +4

$$= -\frac{1}{4} - \frac{1}{2}\lim_{b\to 0^+}\left(\frac{\ln(b)}{\frac{1}{b^2}}\right) \quad \frac{-\infty}{\infty}$$

L'Hopitals

$$= \frac{-1}{4} - \frac{1}{2}\lim_{b\to 0}\left(\frac{\frac{1}{b}}{\frac{-2}{b^3}}\right) = \frac{-1}{4} - \frac{1}{2}\lim_{b\to 0}\frac{-b^2}{2} = \boxed{\frac{-1}{4}}$$

+3 +2

2. (5 points) Determine whether the sequence converges or diverges. If it converges, find the limit.

$$a_n = \frac{n}{2^n}$$

$$\lim_{n\to\infty} a_n = \lim_{n\to\infty}\frac{n}{2^n} = \lim_{n\to\infty}\frac{1}{2^n\ln(2)} = \boxed{0}$$ +3

The sequence converges. +2

2^n grows faster than n.