Math 1552
Spring 2019
Quiz 7 4:30pm Rubric
March 14, 2019
Time Limit: 15 Minutes

Name (Print): ______________________

Canvas email: ______________________

Teaching Assistant/Section: ______________________

GT ID:									

By signing here, you agree to abide by the **Georgia Tech Honor Code**: *I commit to uphold the ideals of honor and integrity by refusing to betray the trust bestowed upon me as a member of the Georgia Tech Community.*

Sign Your Name: ______________________

This quiz contains 2 pages (including this cover page) and 1 problems. Check to see if any pages are missing. Enter all requested information on the top of this page.

You may *not* use your books, notes, or any calculator on this quiz.

You are required to show your work on each problem on this quiz. The following rules apply:

- **If you use a "fundamental theorem" you must indicate this** and explain why the theorem may be applied.
- **Organize your work**, in a reasonably neat and coherent way, in the space provided. Work scattered all over the page without a clear ordering will receive very little credit.
- **Mysterious or unsupported answers will not receive full credit**. A correct answer, unsupported by calculations, explanation, or algebraic work will receive no credit; an incorrect answer supported by substantially correct calculations and explanations might still receive partial credit.
- Please circle or box in your final answer.

Problem	Points	Score
1	20	
Total:	20	

1. Use any test for convergence we have learned thus far to determine if the following series converge or diverge. Be sure to show your work and justify your answer completely.

   (a) (10 points)

$$\sum_{k=2}^{\infty} \frac{\sqrt{k}}{\sqrt{k^2-3}}$$

Solution:

$$\sqrt{k^2-3} < \sqrt{k^2} = k \qquad \to \qquad \frac{\sqrt{k}}{\sqrt{k^2+3}} > \frac{\sqrt{k}}{k} = \frac{1}{\sqrt{k}}$$

The series $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k}}$ is a p-series with $p = 1/2 < 1$, so it diverges. Then by the direct comparison test, $\sum_{k=1}^{\infty} \frac{\sqrt{k}}{\sqrt{k^2-3}}$ also diverges.

   (b) (10 points)

$$\sum_{k=1}^{\infty} ke^{-k^2}$$

Solution: Note that $f(x) = xe^{-x^2}$ is positive, continuous, and decreasing. So we will use the integral test:

$$\int_1^{\infty} xe^{-x^2} dx = \lim_{a\to\infty} \int_1^a xe^{-x^2} dx$$

u-sub: $u = -x^2 \qquad du = -2xdx$

$$= \lim_{a\to\infty} \int_{u=-1}^{-a^2} \frac{-e^u}{2} du = \lim_{a\to\infty} -e^u \Big|_{-1}^{-a^2} = \lim_{a\to\infty} (-e^{-a^2} + e^{-1}) = e^{-1}$$

By the integral test, since the improper integral converges, so does the series.

Point breakdown for both parts:
1 points: Identify the test you are using.
3 points: State conditions for the test (e.g. positive, continuous, decreasing for integral test) OR state a comparison series with its convergence.
4 points: Doing correct work/arithmetic for the test (e.g. limit, inequality, improper integral).
2 points: Make a conlcusion statement about convergenge or divergence and why.