Math 2551 Worksheet 8 - Review for Exam 1

1. Set up the integral to find the arc length of the curve $y = e^x$ from the point $(0, 1)$ to the point $(1, e)$. Focus on finding a parameterization, and on what values of t give these two points. Is this an integral you would want to compute? Why or why not?

2. Parameterize the line tangent to the curve
$$\mathbf{r}(t) = \langle \cos^2(t), \sin(t)\cos(t), \cos(t)\rangle$$
at the point where $t = \pi/2$.

3. Compute the unit tangent vector $\mathbf{T}(t)$ and the unit normal vector $\mathbf{N}(t)$ to the circle
$$\mathbf{r}(t) = \langle 2\cos(t), 2\sin(t)\rangle.$$
Before checking, should the normal vector be pointing into or out of the circle? Why?

4. We have seen that the curvature of a circle with radius a is $1/a$. Thinking about the geometry of a helix with radius a, do you think its curvature will be greater than or less than $1/a$? Why? Compute the curvature using the parameterization
$$\mathbf{r}(t) = \langle a\cos(t), t, a\sin(t)\rangle$$
to confirm or challenge your intuition.

5. The function $\ell(t)$ below describes a line. There is a particular plane that $\ell(t)$ is normal to at the point $t = 0$. Find an equation of this plane.
$$\ell(t) = \langle 3 - 3t, 2 + t, -2t\rangle.$$
Where does this line intersect the different plane $3x - y + 2z = -7$?

6. Find and sketch the domain of each of the following functions of two variables:
   (a) $\sqrt{9 - x^2} + \sqrt{y^2 - 4}$
   (b) $\arcsin(x^2 + y^2 - 2)$
   (c) $\sqrt{16 - x^2 - 4y^2}$

7. Solve the differential equation below, together with its given initial conditions. Remember that this means finding all functions $\mathbf{r}(t)$ which satisfy the given equations.
$$\mathbf{r}''(t) = 2\mathbf{i} + 6t\mathbf{j} + \frac{1}{2\sqrt{t}}\mathbf{k}, \quad \mathbf{r}'(1) = 2\mathbf{i} + 3\mathbf{j} + \mathbf{k}, \quad \mathbf{r}(1) = \mathbf{i} + \mathbf{j}$$

8. Let $f(x, y) = (x^2 - y^2)/(x^2 + y^2)$ for $(x, y) \neq (0, 0)$. Is it possible to define $f(0, 0)$ in a way that makes f continuous at the origin? Why?