Math 2551 Worksheet: Exam 2 Review

1. Which of the following statements are true if $f(x, y)$ is differentiable at (x_0, y_0)? Give reasons for your answers.
   (a) If $\mathbf{u}$ is a unit vector, the derivative of f at (x_0, y_0) in the direction of $\mathbf{u}$ is $(f_x(x_0, y_0)\mathbf{i} + f_y(x_0, y_0)\mathbf{j}) \cdot \mathbf{u}$.
   (b) The derivative of f at (x_0, y_0) in the direction of $\mathbf{u}$ is a vector.
   (c) The directional derivative of f at (x_0, y_0) has its greatest value in the direction of ∇f.
   (d) At (x_0, y_0), the vector ∇f is normal to the curve $f(x, y) = f(x_0, y_0)$.
2. Find dw/dt at $t = 0$ if $w = \sin(xy + \pi), x = e^t$, and $y = \ln(t + 1)$.
3. Find the extreme values of $f(x, y) = x^3 + y^2$ on the circle $x^2 + y^2 = 1$.
4. Test the function $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2$ for local maxima and minima and saddle points and find the function's value at these points.
5. Find the points on the surface $xy + yz + zx - x - z^2 = 0$ where the tangent plane is parallel to the xy-plane.
6. Evaluate the integral $\displaystyle\int_0^1 \int_{2y}^2 4\cos(x^2)\, dx\, dy$. Describe why you made any choices you did in the course of evaluating this integral.
7. If $f(x, y) \geq 2$ for all (x, y), is it possible that the average value of $f(x, y)$ on a unit disk centered at the origin is $\dfrac{2}{\pi}$?
8. A swimming pool is circular with a 40 foot diameter. The depth is constant along east-west lines and increases linearly from 2 feet at the south end to 7 feet at the north end. Find the volume of water in the pool.