Math 2551 Worksheet: Review for Exam 3

1. Set up an iterated integral in spherical coordinates for $\iiint_E z^2 \, dV$ where E is the region between the spheres $x^2+y^2+z^2=4$ and $x^2+y^2+z^2=25$ and inside $z=-\sqrt{\frac{1}{3}(x^2+y^2)}$.

2. Set up an integral that computes the volume of the solid which is bounded above by the cylinder $z=4-x^2$, on the sides by the cylinder $x^2+y^2=4$, and below by the xy-plane using

   (a) Cartesian coordinates

   (b) cylindrical coordinates

   Which integral would you rather evaluate and why?

3. Find an integral that computes the mass of the wire which lies along the curve $y^2=x^3$ from $(0,0)$ to $(1,-1)$ and has density function $\rho(x,y)=2xy^2$.

4. Show that the field $\mathbf{F}=2x\mathbf{i}-y^2\mathbf{j}-\frac{4}{1+z^2}\mathbf{k}$ is conservative, find a potential function, and use it to compute the integral

$$\int_C 2x \, dx - y^2 \, dy - \frac{4}{1+z^2} \, dz$$

   where C is any path from $(0,0,0)$ to $(3,3,1)$.

5. Compute $\int_C (6y+x) \, dx + (y+2x) \, dy$ using any method, where C is the circle $(x-2)^2+(y-3)^2=4$.

6. Find the flux of the field $\mathbf{F}=y\mathbf{i}-x\mathbf{j}+\mathbf{k}$ through the portion of the sphere $x^2+y^2+z^2=a^2$ in the first octant in the direction away from the origin.

7. Use Stokes' theorem to show that the circulation of the field $\mathbf{F}=\langle 2x, 2y, 2z\rangle$ around the boundary curve C of **any** smooth orientable surface S in $\mathbb{R}^3$ is 0.

8. Find the outward flux of $\mathbf{F}=(x\mathbf{i}+y\mathbf{j}+z\mathbf{k})/\sqrt{x^2+y^2+z^2}$ through the boundary S of the "thick sphere" D given by the points satisfying $1 \le x^2+y^2+z^2 \le 4$.