Math 2551 Worksheet: Review for Final

1. Find the equation of the plane through $(1, -1, 3)$ parallel to the plane $3x + y + z = 7$. Is there a unique plane through $(1, -1, 3)$ which is perpendicular to the plane $3x+y+z = 7$. Explain why or why not.

2. Find the point on the curve

$$\mathbf{r}(t) = (5\sin(t))\mathbf{i} + (5\cos(t))\mathbf{j} + 12t\mathbf{k}$$

at a distance 26π units along the curve from the point $(0, 5, 0)$ in the direction of increasing parameter t.

3. Find the domain and range of $f(x, y) = \sqrt{x^2 - y}$ and identify its level curves.

4. Compute $\lim_{(x,y)\to(0,0)} \frac{y}{x^2 - y}$ or show this limit does not exist.

5. Let $f(x, y, z) = xy + 2yz - 3xz$. Find the tangent plane to the surface $f(x, y, z) = 1$ at $(1, 1, 0)$ and the linearization $L(x, y, z)$ at $(1, 1, 0)$.

6. At the point $(1, 2)$, the function $f(x, y)$ has a derivative of 2 in the direction toward $(2, 2)$ and a derivative of -2 in the direction toward $(1, 1)$. Find $\nabla f(1, 2)$ and the derivative of f at $(1, 2)$ in the direction toward the point $(4, 6)$.

7. Find the value of the derivative of $f(x, y, z) = xy+yz+xz$ with respect to t on the curve $\mathbf{r}(t) = \langle\cos(t), \sin(t), \cos(2t)\rangle$ at $t = 1$.

8. Find the local minima, local maxima, and saddle points of the function $f(x, y) = x^4 - 8x^2 + 3y^2 - 6y$.

9. Find the extreme values of $f(x, y) = 4xy - x^4 - y^4 + 16$ on the triangular region bounded below by the line $y = -2$, above by the line $y = x$, and on the right by the line $x = 2$.

10. Find the extreme values of $f(x, y) = xy$ on the circle $x^2 + y^2 = 1$.

11. Sketch the region of integration and reverse the order of integration for the integral

$$\int_0^{3/2} \int_{-\sqrt{9-4y^2}}^{\sqrt{9-4y^2}} y\ dx\ dy.$$

12. Evaluate the integral

$$\int_{-1}^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} \frac{2\ dy\ dx}{(1 + x^2 + y^2)^2}$$

by changing to polar coordinates.

13. Find the centroid of the region bounded by the lines $x = 2, y = 2$, and the hyperbola $xy = 2$ in the xy-plane.

14. Find the volume of the region bounded above by the sphere $x^2 + y^2 + z^2 = 2$ and below by the paraboloid $z = x^2 + y^2$.

15. Use the transformation $u = 3x + 2y, v = x + 4y$ to evaluate the integral

$$\iint_R (3x^2 + 14xy + 8y^2)\ dx\ dy$$

where R is the region in the first quadrant bounded by the lines $y = (-3/2)x + 1, y = (-3/2)x + 3, y = -(1/4)x$, and $y = -(1/4)x + 1)$.

16. Evaluate the integral $\int_C y^2\ dx + x^2\ dy$ where C is the circle $x^2 + y^2 = 4$.

17. Find the outward flux of $\mathbf{F} = 2xy\mathbf{i} + 2yz\mathbf{j} + 2xz\mathbf{k}$ across the boundary of the cube cut from the first octant by the planes $x = 1, y = 1, z = 1$.

18. Find the work done by $\mathbf{F} = \dfrac{x\mathbf{i} + y\mathbf{j}}{(x^2 + y^2)^{3/2}}$ over the plane curve $\mathbf{r}(t) = \langle e^t \cos(t), e^t \sin(t) \rangle$ from the point $(1, 0)$ to the point $(e^{2\pi}, 0)$.

19. Find the flux of the field $\mathbf{F} = \langle 2xy + x, xy - y \rangle$ outward across the boundary of the square bounded by $x = 0, x = 1, y = 0, x = 1$.

20. Find the flux of $\mathbf{F} = xz\mathbf{i} + yz\mathbf{j} + \mathbf{k}$ across the upper cap cut from the sphere $x^2 + y^2 + z^2 = 25$ by the plane $z = 3$, oriented away from the xy-plane.