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```
clc
format shortG
```

Part 1: first data set

data is given by an mx2 matrix with x-values in column 1 and y-values in column 2

```
data1=[ 1 8 ; 2 6 ; 4 5 ; 5 7 ; 8 16 ];
% extracts the x-values in column 1
x=data1(:,1);
% extracts the y-values in column 2
y=data1(:,2);
```

first model: linear $y=a_0+a_1*x$

constructs the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

```
A=[1 x(1) ;
   1 x(2) ;
   1 x(3) ;
   1 x(4) ;
   1 x(5) ]
% b vector is just the y-values
b=y;
% gets xhat least squares solution using fact that A has linearly
% independent columns so A^T*A is invertible - note A' is MATLAB transpose
xhat=inv(A'*A)*A'*b;
% extracts alpha and beta values from the xhat vector
a0=xhat(1);
a1=xhat(2);
% defines the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f1=@(x) a0+a1.*x;
```

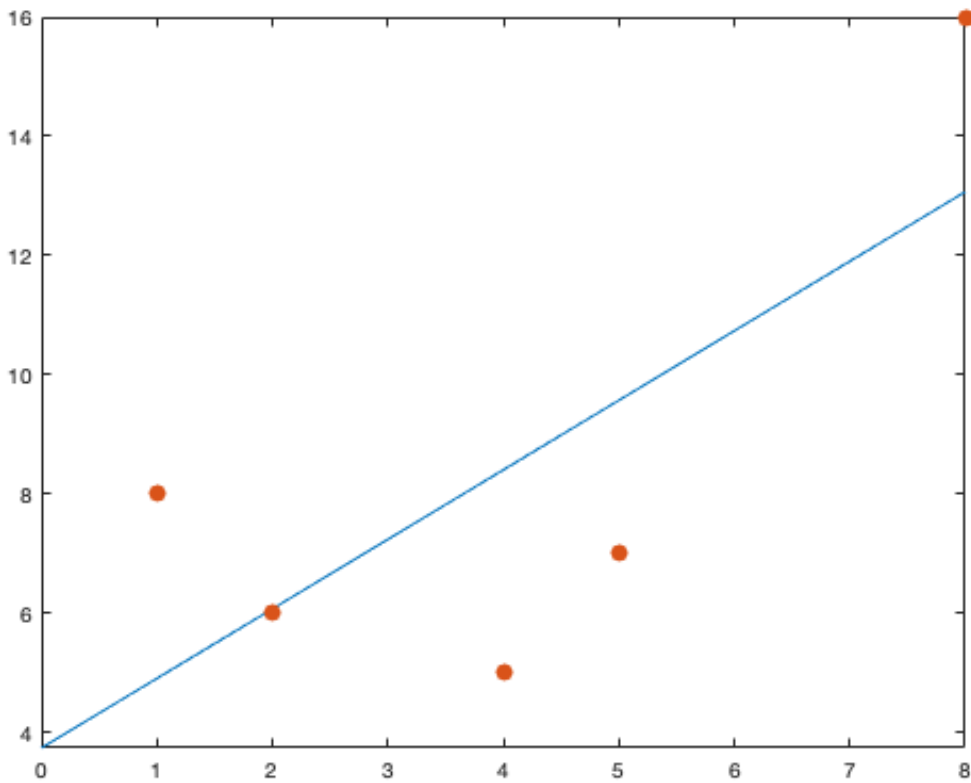
```
% compute the least squares error ||A*xhat-b||  
errorLine=sqrt((A*xhat-b)'*(A*xhat-b))  
% draws figure  
figure(1)  
fplot(f1,[0 8])  
hold on  
plot(x,y,'.','markersize',28)  
hold off
```

A =

```
1    1  
1    2  
1    4  
1    5  
1    8
```

errorLine =

```
6.0305
```



second model: full quadratic $y=a_0+a_1*x+a_2*x^2$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

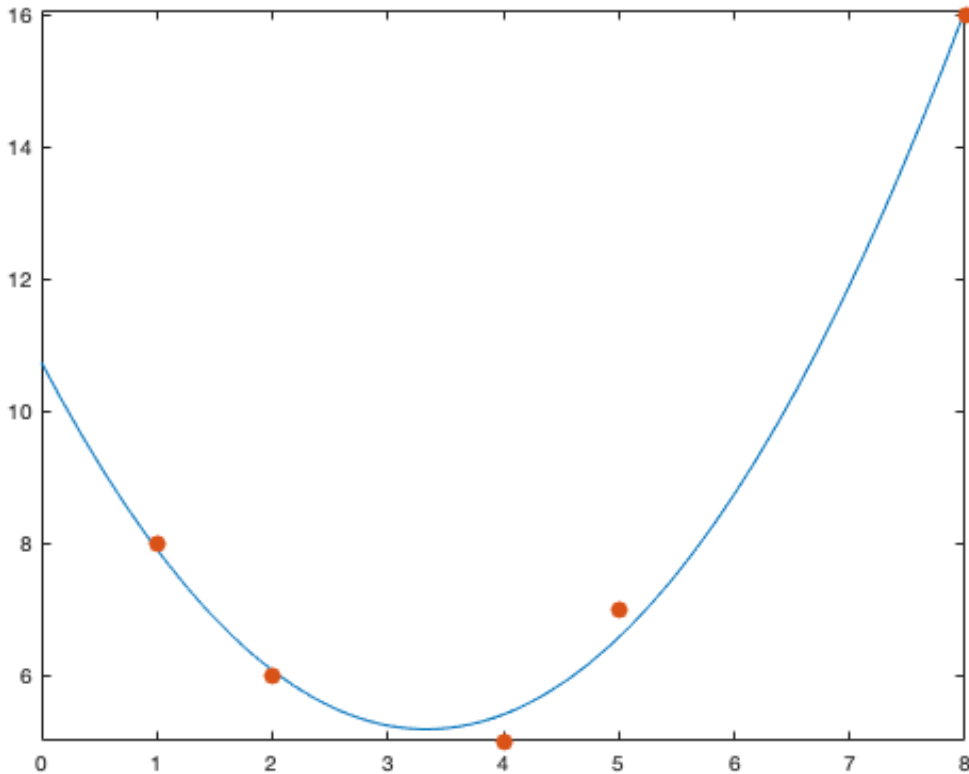
```
A=[1 x(1) x(1)^2 ;
    1 x(2) x(2)^2 ;
    1 x(3) x(3)^2 ;
    1 x(4) x(4)^2 ;
    1 x(5) x(5)^2 ]
% copy-pasted/modified code from part 1
b=y;
xhat=inv(A'*A)*A'*b;
a0=xhat(1);
a1=xhat(2);
a2=xhat(3);
% defines the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f2=@(x) a0+a1.*x+a2.*x.^2;
% least squares error ||A*xhat-b||
errorQuad=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
figure(2)
fplot(f2,[0 8])
hold on
plot(x,y, '.', 'markersize',28)
hold off
```

A =

1	1	1
1	2	4
1	4	16
1	5	25
1	8	64

errorQuad =

0.60553



third model: [STUDENT FILL IN:] linear+cosine $y=a_0*x+a_1*\cos(x)$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model

```
A=[ x(1) cos(x(1)) ;
    x(2) cos(x(2)) ;
    x(3) cos(x(3)) ;
    x(4) cos(x(4)) ;
    x(5) cos(x(5)) ]
b=y;
xhat=inv(A'*A)*A'*b;
% [STUDENT FILL IN:] you may need to add/delete variables depending on how
% many terms your model has
a0=xhat(1);
a1=xhat(2);
% [STUDENT FILL IN:] define the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f3=@(x) a0.*x+a1.*cos(x) ;
% least squares error is ||A*xhat-b||
errorStudentModelforData1=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
figure(3)
fplot(f3,[0 8])
```

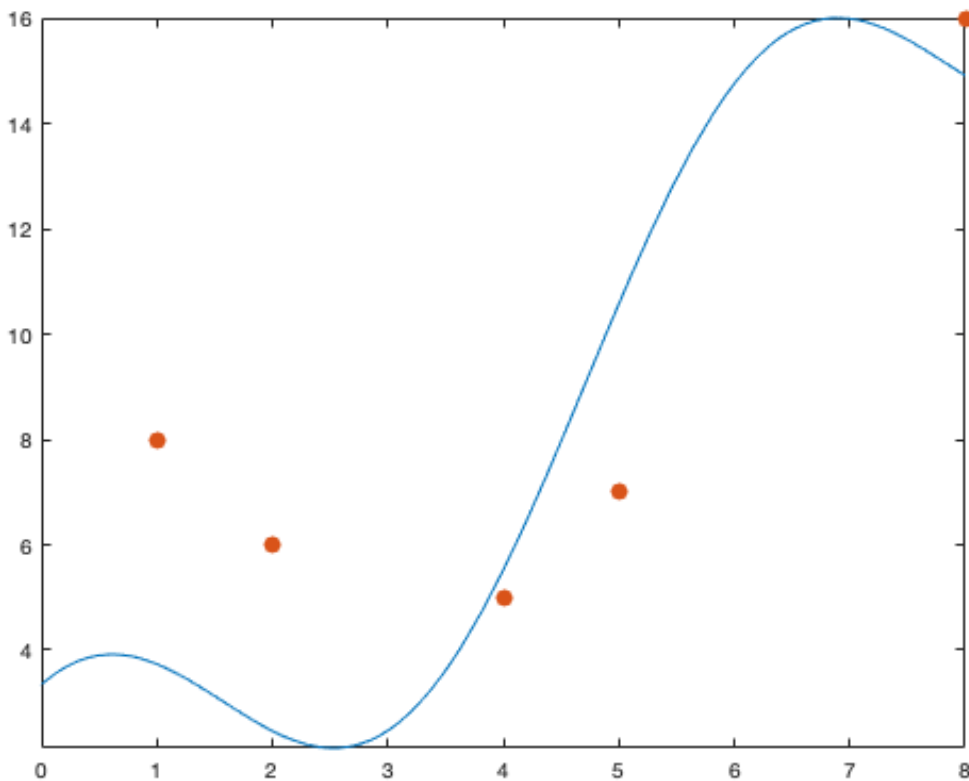
```
hold on
plot(x,y,'.','markersize',28)
hold off
```

A =

1	0.5403
2	-0.41615
4	-0.65364
5	0.28366
8	-0.1455

errorStudentModelforData1 =

6.7064



answer questions: [STUDENT FILL IN:] answer questions as comments

Q1: which model has the best error (smallest error value)? Ans1: Q2: what does the value of the error $|A \cdot \hat{a} - b|$ represent geometrically? Ans2:

Part 2: student created data set must have at least 6 data points

[STUDENT FILL IN:] create a data2 matrix where the data is given by an mx2 matrix with m>5 (at least 6 data points) with x-values in column 1 and y-values in column 2

```
data2=[ 1 1 ; 2 3 ; 4 5 ; 6 7 ; 7 10 ; 8 12 ];
% extracts the x-values in column 1
x=data2(:,1);
% extracts the y-values in column 2
y=data2(:,2);
```

fourth model: [STUDENT FILL IN:] log+exponential $y=a_0\log(x)+a_1\exp(x)$

[STUDENT FILL IN:] student should construct the design matrix which is the coefficient matrix of the system $Ax=b$ obtained from plugging in data points into the model NOTE: do ***not*** use any of the previous models 1 through 3

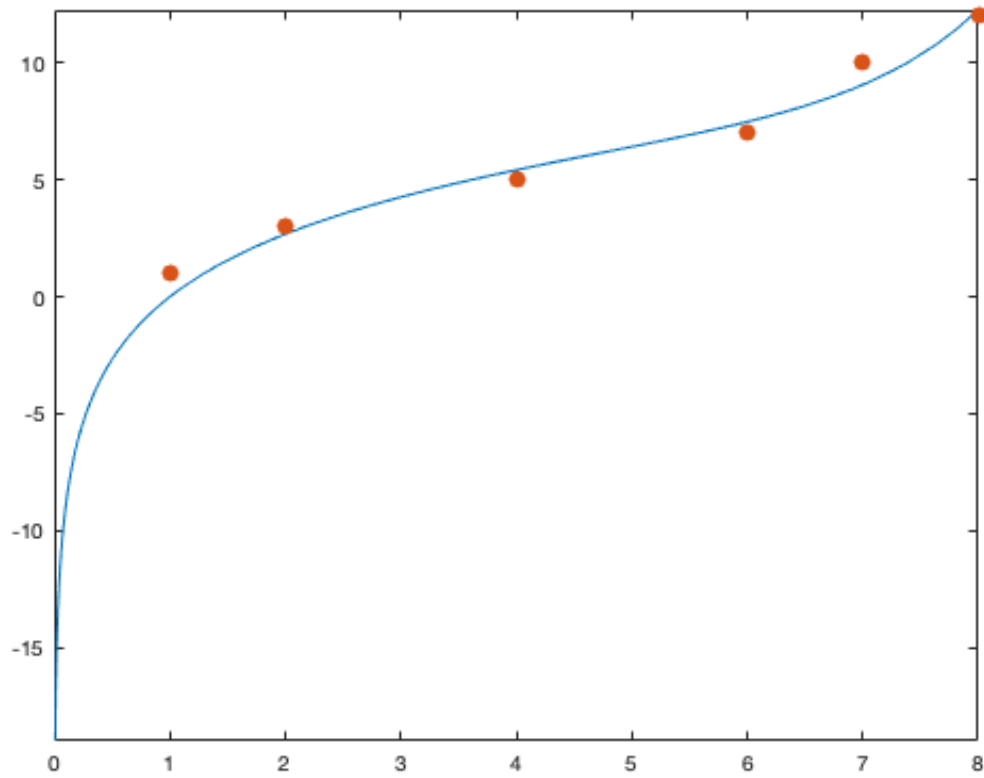
```
A=[ log(x(1)) exp(x(1)) ;
    log(x(2)) exp(x(2)) ;
    log(x(3)) exp(x(3)) ;
    log(x(4)) exp(x(4)) ;
    log(x(5)) exp(x(5)) ;
    log(x(6)) exp(x(6)) ]
b=y;
xhat=inv(A'*A)*A'*b;
% [STUDENT FILL IN:] you may need to add/delete variables depending on how
% many terms your model has
a0=xhat(1);
a1=xhat(2);
% [STUDENT FILL IN:] define the function for plotting
% WARNING: you must use .* for 'times' and .^ for 'exponent' or else
% MATLAB will give an error code
f4=@(x) a0.*log(x)+a1.*exp(x) ;
% least squares error is ||A*xhat-b||
errorStudentModelforData2=sqrt((A*xhat-b)'*(A*xhat-b))
% draws figure
% [STUDENT FILL IN:] You may need to modify the range of the fplot second
% argument [0 8] if your x-values go outside the range 0 <= x <= 8
figure(4)
fplot(f4,[0 8])
hold on
plot(x,y,'.','markersize',28)
hold off
```

```
A =
     0     2.7183
 0.69315  7.3891
 1.3863  54.598
 1.7918  403.43
```

```
1.9459    1096.6  
2.0794    2981
```

errorStudentModelforData2 =

```
1.5649
```



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