UNIT 1 - Polynomial functions, graphing, solving equations

A.1 sets, intervals, absolute value, exponent rules
A.2 factoring, polynomials
A.6 solving quadratic equations, radical expressions
A.8 complex numbers, conjugates

1.1 coordinate plane, quadrants, equation of a circle, distance, midpoint, intercepts, symmetry
1.2 lines, slope, slope-intercept, parallel/perpendicular lines
1.3 functions, domain, range, difference quotient
1.4 increasing, decreasing, max/min, even/odd, piecewise functions
1.5 horizontal/vertical shifts, reflections, stretching/compressing
1.6 composite functions, domain of a composite function
1.7 one-to-one, function inverse, range of one-to-one functions
7.1 solving systems of equations, substitution method, elimination method

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SECTION A.1 Exercises

Basic Concepts and Skills

In Exercises 1–4, write each of the following rational numbers as a decimal and state whether the decimal is repeating or terminating.

1. $-\frac{4}{5}$ **2.** $-\frac{3}{12}$

3. $\frac{3}{11}$ **4.** $\frac{41}{15}$

In Exercises 5–10, classify each of the following numbers as rational or irrational.

5. -207 **6.** $-\sqrt{25}$

7. $\sqrt{32}$ **8.** $5 + \sqrt{18}$

9. 0.321 **10.** $5.8\overline{2}$

In Exercises 11–14, find each set, given $A = \{-4, -2, 0, 2, 4\}$, $B = \{-3, 0, 1, 2, 3, 4\}$, and $C = \{-4, -3, -2, -1, 0, 2\}$.

11. $A \cup B$

12. $A \cap B$

13. $(A \cup B) \cap C$

14. $(A \cup B) \cup C$

In Exercises 15 and 16, convert each decimal to a quotient of two integers in lowest terms.

15. 3.75 **16.** -2.35

In Exercises 17–24, find the union and the intersection of the given intervals.

17. $I_1 = (-2, 3]; I_2 = [1, 5)$

18. $I_1 = [1, 7]; I_2 = (3, 5)$

19. $I_1 = (-6, 2); I_2 = [2, 10)$

20. $I_1 = (-\infty, -3]; I_2 = (-3, \infty)$

21. $I_1 = (-\infty, 7); I_2 = (-\infty, 3)$

22. $I_1 = (-2, \infty); I_2 = (0, \infty)$

23. $I_1 = (-\infty, 10); I_2 = (10, \infty)$

24. $I_1 = (-5, 6); I_2 = (-\infty, \infty)$

In Exercises 25–38, rewrite each expression without absolute value bars.

25. $|20|$ **26.** $|12|$

27. $-|-4|$ **28.** $-|-17|$

29. $\left|\frac{5}{7}\right|$ **30.** $\left|\frac{-3}{5}\right|$

31. $|5 - \sqrt{2}|$ **32.** $|\sqrt{2} - 5|$

33. $\frac{8}{|-8|}$ **34.** $\frac{-8}{|8|}$

35. $|5 + |-7||$ **36.** $|5 - |-7||$

37. $||7| - |4||$ **38.** $||4| - |7||$

In Exercises 39–46, use the absolute value to express the distance between the points with coordinates a and b on the number line. Then determine this distance by evaluating the absolute value expression.

39. $a = 3$ and $b = 8$ **40.** $a = 2$ and $b = 14$

41. $a = -6$ and $b = 9$ **42.** $a = -12$ and $b = 3$

43. $a = -20$ and $b = -6$ **44.** $a = -14$ and $b = -1$

45. $a = \frac{22}{7}$ and $b = -\frac{4}{7}$ **46.** $a = \frac{16}{5}$ and $b = -\frac{3}{5}$

In Exercises 47–58, graph each of the given intervals on a separate number line and write the inequality notation for each.

47. $[1, 4]$ **48.** $[-2, 2]$

49. $(14, 28)$ **50.** $\left(\frac{1}{2}, \frac{9}{2}\right)$

51. $(-3, 1]$ **52.** $[-6, -2)$

53. $[-3, \infty)$ **54.** $[0, \infty)$

55. $(-\infty, 5]$ **56.** $(-\infty, -1]$

57. $\left(-\frac{3}{4}, \frac{9}{4}\right)$ **58.** $\left(-3, -\frac{1}{2}\right)$

In Exercises 59–68, evaluate each expression for $x = 3$ and $y = -5$.

59. $2(x + y) - 3y$ **60.** $-2(x + y) + 5y$

61. $3|x| - 2|y|$ **62.** $7|x - y|$

63. $\frac{x - 3y}{2} + xy$ **64.** $\frac{y + 3}{x} - xy$

65. $\frac{2(1 - 2x)}{y} - (-x)y$ **66.** $\frac{3(2 - x)}{y} - (1 - xy)$

67. $\dfrac{\frac{14}{x} + \frac{1}{2}}{\frac{-y}{4}}$ **68.** $\dfrac{\frac{4}{-y} + \frac{8}{x}}{\frac{y}{2}}$

In Exercises 69–78, name the exponent and the base.

69. 17^3

70. 10^2

71. 9^0

72. $(-2)^0$

73. $(-5)^5$

74. $(-99)^2$

75. -10^3

76. $-(-3)^7$

77. a^2

78. $(-b)^3$

In Exercises 79–100, evaluate each expression.

79. 6^1 **80.** 3^4

81. 7^0 **82.** $(-8)^0$

83. $(2^3)^2$ **84.** $(3^2)^3$

85. $(3^2)^{-2}$ **86.** $(7^2)^{-1}$

87. $(5^{-2})^3$ **88.** $(5^{-1})^3$

89. $(4^{-3}) \cdot (4^5)$ **90.** $(7^{-2}) \cdot (7^3)$

91. $3^{-2} + \left(\frac{1}{3}\right)^2$ **92.** $5^{-2} + \left(\frac{1}{5}\right)^2$

93. $\frac{2^{11}}{2^{10}}$ **94.** $\frac{3^6}{3^8}$

95. $\frac{(5^3)^4}{5^{12}}$ **96.** $\frac{(9^5)^2}{9^8}$

97. $\frac{2^5 \cdot 3^{-2}}{2^4 \cdot 3^{-3}}$ **98.** $\frac{4^{-2} \cdot 5^3}{4^{-3} \cdot 5}$

99. $\left(\frac{11}{7}\right)^{-2}$ **100.** $\left(\frac{13}{5}\right)^{-2}$

In Exercises 101–134, simplify each expression. Write your answers without negative exponents. Whenever an exponent is negative or zero, assume that the base is not zero.

101. x^4y^0 **102.** $x^{-1}y^0$

103. $x^{-1}y$ **104.** x^2y^{-2}

105. $-8x^{-1}$ **106.** $(-8x)^{-1}$

107. $x^{-1}(3y^0)$ **108.** $x^{-3}(3y^2)$

109. $x^{-1}y^{-2}$

110. $x^{-3}y^{-2}$

111. $(x^{-3})^4$

112. $(x^{-5})^2$

113. $(x^{-11})^{-3}$

114. $(x^{-4})^{-12}$

115. $-3(xy)^5$

116. $-8(xy)^6$

117. $4(xy^{-1})^2$

118. $6(x^{-1}y)^3$

119. $3(x^{-1}y)^{-5}$

120. $-5(xy^{-1})^{-6}$

121. $\frac{(x^3)^2}{(x^2)^5}$

122. $\frac{x^2}{(x^3)^4}$

123. $\left(\frac{2xy}{x}\right)^3$

124. $\left(\frac{5xy}{x^3}\right)^4$

125. $\left(\frac{-3x^2y}{x}\right)^5$

126. $\left(\frac{-2xy^2}{y}\right)^3$

127. $\left(\frac{-3x}{5}\right)^{-2}$

128. $\left(\frac{-5y}{3}\right)^{-4}$

129. $\left(\frac{4x^{-2}}{xy^5}\right)^3$

130. $\left(\frac{3x^2y}{y^3}\right)^5$

131. $\frac{x^3y^{-3}}{x^{-2}y}$

132. $\frac{x^2y^{-2}}{x^{-1}y^2}$

133. $\frac{27x^{-3}y^5}{9x^{-4}y^7}$

134. $\frac{15x^5y^{-2}}{3x^7y^{-3}}$

Applying the Concepts

135. **Media players.** Let A = the set of people who own MP3 players and B = the set of people who own DVD players.
a. Describe the set $A \cup B$.
b. Describe the set $A \cap B$.

136. **Standard car features.** The table indicates whether certain features are "standard" for each of three types of car.

	Navigation System	Automatic Transmission	Leather Seats
2016 Lexus GS 350	yes	yes	yes
2016 Lincoln MKZ	no	yes	yes
2016 Cadillac ATS-V	no	no	yes

Use the roster method to describe each of the following sets.

a. A = cars in which a navigation system is standard.
b. B = cars in which an automatic transmission is standard.
c. C = cars in which leather seats are standard.
d. $A \cap B$
e. $B \cap C$
f. $A \cup B$
g. $A \cup C$

140. **Downloading music.** To download a 4 MB song with a 56 Kbs modem takes an average of 15 minutes. Use absolute value notation to write an expression that describes the difference between this average time and the actual time it took to download the following songs. Then evaluate that expression.
a. *Believe* (Cher): 14 minutes
b. *Caged Bird* (Alicia Keys): 17.5 minutes
c. *Somewhere* (Barbra Streisand): 15 minutes

142. **Circle area.** The area A of a circle with diameter d is given by $A = \pi\left(\frac{d}{2}\right)^2$. Use this relationship to
a. verify that doubling the length of the diameter of a circular skating rink increases the area of the rink by a factor of 2^2.
b. verify that tripling the length of the diameter of a circular skating rink increases the area of the rink by a factor of 3^2.

SECTION A.2 Exercises

Basic Concepts and Skills

In Exercises 1–4, determine whether the given expression is a polynomial. If it is, write it in standard form.

1. $1 + x^2 + 2x$

2. $x - \frac{1}{x}$

3. $x^{-2} + 3x + 5$

4. $3x^4 + x^7 + 3x^5 - 2x + 1$

In Exercises 5–8, find the degree and list the terms of the polynomial.

5. $7x + 3$

6. $-3x^2 + 7$

7. $x^2 - x^4 + 2x - 9$

8. $x + 2x^3 + 9x^7 - 21$

In Exercises 9–16, perform the indicated operations. Write the resulting polynomial in standard form.

9. $(x^3 + 2x^2 - 5x + 3) + (-x^3 + 2x - 4)$

10. $(x^3 - 3x + 1) + (x^3 - x^2 + x - 3)$

11. $(2x^3 - x^2 + x - 5) - (x^3 - 4x + 3)$

12. $(-x^3 + 2x - 4) - (x^3 + 3x^2 - 7x + 2)$

13. $-2(3x^2 + x + 1) + 6(-3x^2 - 2x - 2)$

14. $2(5x^2 - x + 3) - 4(3x^2 + 7x + 1)$

15. $(3y^3 - 4y + 2) + (2y + 1) - (y^3 - y^2 + 4)$

16. $(5y^2 + 3y - 1) - (y^2 - 2y + 3) + (2y^2 + y + 5)$

In Exercises 17–50, perform the indicated operations.

17. $6x(2x + 3)$

18. $7x(3x - 4)$

19. $(x + 1)(x^2 + 2x + 2)$

20. $(x - 5)(2x^2 - 3x + 1)$

21. $(3x - 2)(x^2 - x + 1)$

22. $(2x + 1)(x^2 - 3x + 4)$

23. $(x + 1)(x + 2)$

24. $(x + 2)(x + 3)$

25. $(3x + 2)(3x + 1)$

26. $(x + 3)(2x + 5)$

27. $(-4x + 5)(x + 3)$

28. $(-2x + 1)(x - 5)$

29. $(3x - 2)(2x - 1)$

30. $(x - 1)(5x - 3)$

31. $(2x - 3a)(2x + 5a)$

32. $(5x - 2a)(x + 5a)$

33. $(x + 2)^2 - x^2$

34. $(x - 3)^2 - x^2$

35. $(x + 3)^3 - x^3$

36. $(x - 2)^3 - x^3$

37. $(4x + 1)^2$

38. $(3x + 2)^2$

39. $(3x + 1)^3$

40. $(2x + 3)^3$

41. $(5 - 2x)(5 + 2x)$

42. $(3 - 4x)(3 + 4x)$

43. $\left(x + \frac{3}{4}\right)^2$

44. $\left(x + \frac{2}{5}\right)^2$

45. $(2x - 3)(x^2 - 3x + 5)$

46. $(x - 2)(x^2 - 4x - 3)$

47. $(1 + y)(1 - y + y^2)$

48. $(y + 4)(y^2 - 4y + 16)$

49. $(x - 6)(x^2 + 6x + 36)$

50. $(x - 1)(x^2 + x + 1)$

In Exercises 51–60, perform the indicated operations.

51. $(x + 2y)(3x + 5y)$

52. $(2x + y)(7x + 2y)$

53. $(2x - y)(3x + 7y)$

54. $(x - 3y)(2x + 5y)$

55. $(x - y)^2(x + y)^2$

56. $(2x + y)^2(2x - y)^2$

57. $(x + y)(x - 2y)^2$

58. $(x - y)(x + 2y)^2$

59. $(x - 2y)^3(x + 2y)$

60. $(2x + y)^3(2x - y)$

In Exercises 61–108, factor each polynomial completely. If a polynomial cannot be factored, state that it is irreducible.

61. $3x^3 - x^2$

62. $2x^3 + 2x^2$

63. $x^2 + 7x + 12$

64. $x^2 + 8x + 15$

65. $x^2 - 6x + 8$

66. $x^2 - 9x + 14$

67. $6x^2 + 17x + 12$

68. $8x^2 - 10x - 3$

69. $x^2 + 6x + 9$

70. $x^2 + 8x + 16$

71. $9x^2 + 6x + 1$

72. $36x^2 + 12x + 1$

73. $x^2 - 64$

74. $x^2 - 121$

75. $16x^2 - 9$

76. $25x^2 - 49$

77. $x^3 - 27$

78. $x^3 - 216$

79. $8 - x^3$

80. $27 - x^3$

81. $x^3 - 3x^2 + x - 3$

82. $x^3 + 5x^2 + x + 5$

83. $x^3 - 5x^2 + x - 5$

84. $x^3 - 7x^2 + x - 7$

85. $x^4 - 1$

86. $x^4 - 81$

87. $20x^4 - 5$

88. $12x^4 - 75$

89. $1 - 16x^2$

90. $4 - 25x^2$

91. $x^2 - 6x + 9$

92. $x^2 - 8x + 16$

93. $4x^2 + 4x + 1$

94. $16x^2 + 8x + 1$

95. $2x^2 - 8x - 10$

96. $5x^2 - 10x - 40$

97. $2x^2 + 3x - 20$

98. $2x^2 - 7x - 30$

99. $x^2 - 12x + 36$

100. $x^2 - 20x + 25$

101. $3x^5 + 12x^4 + 12x^3$

102. $2x^5 + 16x^4 + 32x$

103. $9x^2 - 1$

104. $16x^2 - 25$

105. $16x^2 + 24x + 9$

106. $4x^2 + 20x + 25$

107. $x^2 + 15$

108. $x^2 + 24$

SECTION A.6 Exercises

Basic Concepts and Skills

In Exercises 1–46, find the solution set of each equation. If the equation has no solution write ∅, and if the equation is an identity so state.

1. $5 - (6y + 9) + 2y = 2(y + 1)$

2. $3(4y - 3) = 4[y - (4y - 3)]$

3. $2(3x + 4) = 6(x + 2) - 4$

4. $2(x - 2) + 3x = 6x + 7 - (x - 3)$

5. $\dfrac{2x + 1}{9} - \dfrac{x + 4}{6} = 1$

6. $\dfrac{2 - 3x}{7} + \dfrac{x - 1}{3} = \dfrac{3x}{7}$

7. $\dfrac{1}{3x} + \dfrac{1}{2x} = \dfrac{1}{6} - \dfrac{1}{x}$

8. $\dfrac{2}{x - 1} = \dfrac{3}{x + 1}$

9. $\dfrac{t}{t - 2} = -\dfrac{2}{3}$

10. $\dfrac{2}{x + 1} + 3 = \dfrac{8}{x + 1}$

11. $\dfrac{5x}{x - 1} = \dfrac{5}{x - 1} + 3$

12. $\dfrac{3x}{x + 1} - 3 + \dfrac{3}{2(x + 1)} = 0$

13. $\dfrac{1 + x}{x} = \dfrac{1}{x} + 1$

14. $\dfrac{x - 1}{x - 2} - 1 = \dfrac{1}{x - 2}$

15. $x^2 - 5x = 0$

16. $x^2 - 5x + 4 = 0$

17. $x^2 = 5x + 6$

18. $x = x^2 - 12$

19. $x^2 + 2x - 5 = 0$

20. $x^2 - x - 3 = 0$

21. $3(t^2 - 1) = 2t^2 + 4t + 1$

22. $8(y^2 - y) = y^2 + 3$

23. $x^3 = 2x^2$

24. $3x^4 - 27x^2 = 0$

25. $x^4 - x^3 = x^2 - x$

26. $x^3 - 36x = 16(x - 6)$

27. $\dfrac{x + 3}{x - 1} + \dfrac{x + 4}{x + 1} = \dfrac{8x + 5}{x^2 - 1}$

28. $\dfrac{6}{2x - 2} - \dfrac{1}{x + 1} = \dfrac{2}{2x + 2} + 1$

29. $\dfrac{x}{x - 4} - \dfrac{4}{x + 4} = \dfrac{2x}{x^2 - 16}$

30. $\dfrac{x}{x - 3} + \dfrac{3}{x + 3} = \dfrac{6x}{x^2 - 9}$

31. $\dfrac{1}{x - 1} + \dfrac{x}{x + 3} = \dfrac{4}{x^2 + 2x - 3}$

32. $\dfrac{2x}{x + 3} - \dfrac{x}{x - 1} = \dfrac{14}{x^2 + 2x - 3}$

33. $t - \sqrt{3t + 6} = -2$

34. $\sqrt{5y^2 - 10y + 9} = 2y - 1$

35. $\sqrt{6t - 11} = 2t - 7$

36. $\sqrt{3x + 1} = x - 1$

37. $\sqrt{x - 3} = \sqrt{2x - 5} - 1$

38. $\sqrt{7x + 1} - \sqrt{5x + 4} = 1$

39. $x^{2/3} - 6x^{1/3} + 8 = 0$

40. $x^{2/5} + x^{1/5} - 2 = 0$

41. $2x^{1/2} - x^{1/4} - 1 = 0$

42. $81y^4 + 1 = 18y^2$

43. $(x^2 - 4)^2 - 3(x^2 - 4) - 4 = 0$

44. $(x^2 - 4x)^2 + 7(x^2 - 4x) + 12 = 0$

45. $\left(\dfrac{x}{x + 1}\right)^2 - \dfrac{2x}{x + 1} - 8 = 0$

46. $8\sqrt{\dfrac{x}{x + 3}} - \sqrt{\dfrac{x + 3}{x}} = 2$

47. Solve for x: $\dfrac{x + b}{x - b} = \dfrac{x - 5b}{2x - 5b}$

48. If $a > 0$ solve for x: $3a - \sqrt{ax} = \sqrt{a(3a + x)}$

49. Solve for x: $x = \sqrt{1 + \sqrt{1 + \sqrt{1 + \cdots}}}$

(*Hint*: $x = \sqrt{1 + x}$)

50. Solve for x: $x = 2 + \cfrac{1}{2 + \cfrac{1}{2 + \cdots}}$

$\left(\textit{Hint}: x = 2 + \dfrac{1}{x}\right)$

In Exercises 51 and 52, find the values of k for which the given equation has equal roots.

51. $2x^2 + kx + k = 0$

52. $x^2 + k^2 = 2(k + 1)x$

53. If r and s are the roots of the quadratic equation $ax^2 + bx + c = 0$, show that

$$r + s = -\frac{b}{a} \quad \text{and} \quad r \cdot s = \frac{c}{a}.$$

SECTION A.8 Exercises

Basic Concepts and Skills

In Exercises 1–4, use the definition of equality of complex numbers to find the real numbers x and y such that the equation is true.

1. $2 + xi = y + 3i$

2. $x - 2i = 7 + yi$

3. $x - \sqrt{-16} = 2 + yi$

4. $3 + yi = x - \sqrt{-25}$

In Exercises 5–22, perform each operation and write the result in the standard form $a + bi$.

5. $(5 + 2i) + (3 + i)$

6. $(4 - 3i) - (5 + 3i)$

7. $(3 - 5i) - (3 + 2i)$

8. $(-2 - 3i) + (-3 - 2i)$

9. $3(5 + 2i)$

10. $-4(2 - 3i)$

11. $3i(5 + i)$

12. $-3i(5 - 2i)$

13. $(3 + i)(2 + 3i)$

14. $(4 + 3i)(2 + 5i)$

15. $(2 - 3i)(2 + 3i)$

16. $(4 - 3i)(4 + 3i)$

17. $(3 + 4i)(4 - 3i)$

18. $(-2 + 3i)(-3 + 10i)$

19. $(\sqrt{3} - 12i)^2$

20. $(-\sqrt{5} - 13i)^2$

21. $(1 + 3i)^3$

22. $(1 - 2i)^3$

In Exercises 23–28, write the conjugate $\bar{z}$ of each complex number z. Then find $z\bar{z}$.

23. $z = 2 - 3i$

24. $z = \frac{1}{2} - 2i$

25. $z = 4 + 5i$

26. $z = \frac{2}{3} + \frac{1}{2}i$

27. $z = \sqrt{2} - 3i$

28. $z = \sqrt{5} + \sqrt{3}i$

In Exercises 29–34, write each quotient in the standard form $a + bi$.

29. $\frac{5}{-i}$

30. $\frac{-1}{1 + i}$

31. $\frac{5i}{2 + i}$

32. $\frac{2 + 3i}{1 + i}$

33. $\frac{2 - 5i}{4 - 7i}$

34. $\frac{2 + \sqrt{-4}}{1 + i}$

In Exercises 35–38, find each power of i and simplify the expression.

35. i^{17}

36. i^{125}

37. i^{-7}

38. i^{-24}

In Exercises 39–42, let $z_1 = a + bi$ and $z_2 = c + di$.

39. Prove that $\overline{\overline{z_1}} = z_1$.

40. Prove that $\overline{z_1 + z_2} = \overline{z_1} + \overline{z_2}$.

41. Prove that $\overline{z_1 z_2} = \overline{z_1}\,\overline{z_2}$. Use this fact to prove that $\overline{z^2} = (\bar{z})^2$.

42. Prove that $z_1 + \overline{z_1} = 2a$ and that $z_1 - \overline{z_1} = 2bi$.

SECTION 1.1 Exercises

Concepts and Vocabulary

1. The graph of an equation in two variables such as x and y is the set of all ordered pairs (a, b) ________________.
2. The distance between the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $d(P, Q) =$ ____________________.
3. The coordinates of the midpoint $M = (x, y)$ of the line segment joining $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ are given by $M =$ __________________.
4. The standard form of the equation of a circle with center (h, k) and radius r is ____________________.
5. **True or False.** A point with a negative first coordinate and a positive second coordinate lies in the fourth quadrant.
6. **True or False.** For any points (x_1, y_1) and (x_2, y_2): $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
7. **True or False.** The center of the circle given by the equation $(x + 3)^2 + (y + 4)^2 = 9$ is the point (3,4).
8. **True or False.** If $(-2, 4)$ is a point on a graph that is symmetric with respect to the y-axis, then the point $(2, 4)$ is also on the graph.

In Exercises 13–20, find (a) the distance between P and Q and (b) the coordinates of the midpoint of the line segment PQ.

13. $P(2, 1), Q(3, 5)$
14. $P(1, 4), Q(3, 2)$
15. $P(4, 5), Q(1, -2)$
16. $P(2, 3), Q(-1, 2)$
17. $P(-1, -5), Q(2, -3)$

In Exercises 37–46, graph each equation by plotting points. Let $x = -3, -2, -1, 0, 1, 2$, and 3, where applicable.

37. $y = x + 1$
38. $y = 2x - 1$
39. $y = |x| + 1$
40. $y = |x + 1|$
41. $y = 4 - x^2$
42. $y = x^2 - 4$
43. $y = \sqrt{9 - x^2}$
44. $y = -\sqrt{9 - x^2}$
45. $y = x^3$
46. $y = -x^3$

SECTION 1.2 Exercises

Concepts and Vocabulary

1. The slope of a horizontal line is ______________; the slope of vertical line is ______________ .
2. The slope of the line passing through the points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ is given by the formula $m =$ ______________ .
3. Every line parallel to the line $y = 3x - 2$ has slope, m, equal to ______________ .
4. Every line perpendicular to the line $y = 3x - 2$ has slope, m, equal to ______________ .
5. **True or False.** The slope of the line $y = -\frac{1}{4}x + 5$ is equal to $\frac{1}{4}$.
6. **True or False.** The y-intercept of the line $y = 2x - 3$ is equal to 3.
7. **True or False.** The graph of the line $y = 4$ is a horizontal line.
8. **True or False.** The graph of the line $x = -5$ is a vertical line.

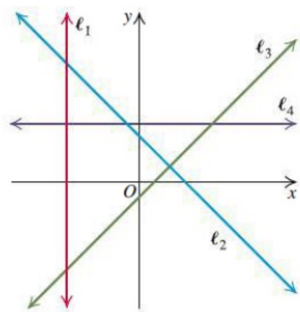
Building Skills

In Exercises 9–16, find the slope of the line through the given pair of points. Without plotting any points, state whether the line is rising, falling, horizontal, or vertical.

9. $(1, 3), (4, 7)$
10. $(0, 4), (2, 0)$
11. $(3, -2), (6, -2)$
12. $(-3, 7), (-3, -4)$
13. $(0.5, 2), (3, -3.5)$
14. $(3, -2), (2, -3)$
15. $(\sqrt{2}, 1), (1 + \sqrt{2}, 5)$
16. $(1 - \sqrt{3}, 0), (1 + \sqrt{3}, 3\sqrt{3})$

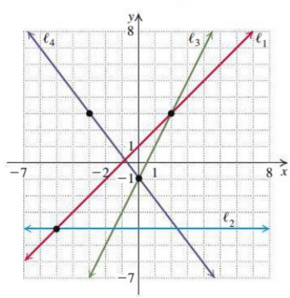
In the figure, identify the line with the given slope m.

17. $m = 1$
18. $m = -1$
19. $m = 0$
20. m is undefined.



In the figure below, find the slope of each line. (The scale is the same on both axes.)

21. ℓ_1
22. ℓ_2
23. ℓ_3
24. ℓ_4



In Exercises 25–32, find an equation in slope–intercept form of the line that passes through the given point and has slope *m*. Also sketch the graph of the line by locating the second point with the rise-and-run method.

25. $(0, 5); m = 3$

26. $(0, 9); m = -2$

27. $(0, 4); m = \frac{1}{2}$

28. $(0, 4); m = -\frac{1}{2}$

29. $(2, 1); m = -\frac{3}{2}$

30. $(-1, 0); m = \frac{2}{5}$

31. $(5, -4); m = 0$

32. $(5, -4)$; m is undefined.

In Exercises 33–54, use the given conditions to find an equation in slope–intercept form of each nonvertical line. Write vertical lines in the form $x = h$.

33. Passing through $(0, 1)$ and $(1, 0)$

34. Passing through $(0, 1)$ and $(1, 3)$

35. Passing through $(-1, 3)$ and $(3, 3)$

36. Passing through $(-5, 1)$ and $(2, 7)$

37. Passing through $(-2, -1)$ and $(1, 1)$

38. Passing through $(-1, -3)$ and $(6, -9)$

39. Passing through $\left(\frac{1}{2}, \frac{1}{4}\right)$ and $(0, 2)$

40. Passing through $(4, -7)$ and $(4, 3)$

41. A vertical line through $(5, 1.7)$

42. A horizontal line through $(1.4, 1.5)$

43. A horizontal line through $(0, 0)$

44. A vertical line through $(0, 0)$

45. $m = 0$; y-intercept $= 14$

46. $m = 2$; y-intercept $= 5$

47. $m = -\frac{2}{3}$; y-intercept $= -4$

48. $m = -6$; y-intercept $= -3$

49. x-intercept $= -3$; y-intercept $= 4$

50. x-intercept $= -5$; y-intercept $= -2$

51. Parallel to $y = 5$; passing through $(4, 7)$

52. Parallel to $x = 5$; passing through $(4, 7)$

53. Perpendicular to $x = -4$; passing through $(-3, -5)$

54. Perpendicular to $y = -4$; passing through $(-3, -5)$

In Exercises 55–64, find the slope and intercepts from the equation of the line. Sketch the graph of each equation.

55. $y = 3x - 2$

56. $y = -2x + 3$

57. $x + 2y - 4 = 0$

58. $x = 3y - 9$

59. $3x - 2y + 6 = 0$

60. $2x = -4y + 15$

61. $x - 5 = 0$

62. $2y + 5 = 0$

63. $x = 0$

64. $y = 0$

In Exercises 65–68, use the two-intercept form of the equation of a line.

65. Find an equation of the line whose x-intercept is 4 and y-intercept is 3.

66. Find an equation of the line whose x-intercept is -3 and y-intercept is 2.

67. Find the x- and y-intercepts of the graph of the equation $2x + 3y = 6$.

68. Repeat Exercise 67 for the equation $3x - 4y + 12 = 0$.

In Exercises 69–72, find the *x* and *y*-intercepts and sketch the graph of the equation

69. $2x - 3y = 12$

70. $3x - 4y = 12$

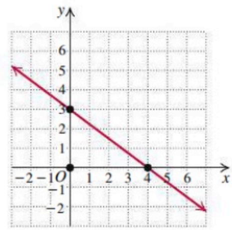
71. $-5x + 2y = 10$

72. $-4x + 5y = 20$

73. Find an equation of the line passing through the points $(2, 4)$ and $(7, 9)$. Use the equation to show that the three points $(2, 4)$ and $(7, 9)$, and $(-1, 1)$ are on the same line (collinear).

74. Use the technique of Exercise 73 to check whether the points $(7, 2)$, $(2, -3)$, and $(5, 1)$ are on the same line.

75. Write the slope–intercept form of the equation of the line that passes through the origin and is parallel to the red line shown in the figure.



76. Write the slope–intercept form of the equation of the line that passes through the origin and is perpendicular to the red line shown in the figure in Exercise 75.

SECTION 1.3 Exercises

Concepts and Vocabulary

1. In the functional notation $y = f(x)$, x is the ______________ variable.
2. If $f(-2) = 7$, then -2 is in the ______________ of the function f and 7 is in the ______________ of f.
3. If the point $(9, -14)$ is on the graph of a function f, then $f(9) =$ ______________.
4. The average rate of change of f as x changes from $x = a$ to $x = b$ is ______________, $a \neq b$.
5. **True or False.** Every relation is a function.
6. **True or False.** If no horizontal line intersects the graph of a relation at more than one point, then the graph of the relation is the graph of a function.
7. **True or False.** The range of a function is the set of all values assigned to the elements in the domain of a function.
8. **True or False.** The average rate of change of a linear function is equal to its slope.

In Exercises 15–28, determine whether each equation defines y as a function of x.

15. $x + y = 2$
16. $x = y - 1$
17. $y = \frac{1}{x}$
18. $xy = -1$
19. $x = |y|$
20. $x = |y - 1|$
21. $y = \frac{1}{\sqrt{2x - 5}}$
22. $y = \frac{1}{\sqrt{x^2 - 1}}$
23. $2 - y = 3x$
24. $3x - 5y = 15$
25. $x^2 + y^2 = 8$
26. $x = y^2$
27. $x^2 + y^3 = 5$
28. $x + y^3 = 8$

In Exercises 29–32, let $f(x) = x^2 - 3x + 1$, $g(x) = \frac{2}{\sqrt{x}}$, and $h(x) = \sqrt{2 - x}$.

29. Find $f(0)$, $g(0)$, $h(0)$, $f(a)$, and $f(-x)$.
30. Find $f(1)$, $g(1)$, $h(1)$, $g(a)$, and $g(x^2)$.
31. Find $f(-1)$, $g(-1)$, $h(-1)$, $h(c)$, and $h(-x)$.
32. Find $f(4)$, $g(4)$, $h(4)$, $g(2 + k)$, and $f(a + k)$.
33. Let $f(x) = \frac{2x}{\sqrt{4 - x^2}}$. Find each function value.
    - **a.** $f(0)$
    - **b.** $f(1)$
    - **c.** $f(2)$
    - **d.** $f(-2)$
    - **e.** $f(-x)$
34. Let $g(x) = 2x + \sqrt{x^2 - 4}$. Find each function value.
    - **a.** $g(0)$
    - **b.** $g(1)$
    - **c.** $g(2)$
    - **d.** $g(-3)$
    - **e.** $g(-x)$

In Exercises 37–52, find the domain of each function.

37. $f(x) = -8x + 7$

38. $f(x) = 2x^2 - 11$

39. $f(x) = \frac{1}{x - 9}$

40. $f(x) = \frac{1}{x + 9}$

41. $h(x) = \frac{2x}{x^2 - 1}$

42. $h(x) = \frac{x - 3}{x^2 - 4}$

43. $G(x) = \frac{3x}{\sqrt{4 - x}}$

44. $f(x) = \frac{-2x}{\sqrt{x + 2}}$

45. $h(x) = \frac{\sqrt{x - 1}}{x - 2}$

46. $H(x) = \frac{\sqrt{2 - x}}{x - 1}$

47. $F(x) = \frac{x + 4}{x^2 + 3x + 2}$

48. $F(x) = \frac{1 - x}{x^2 + 5x + 6}$

49. $g(x) = \frac{\sqrt{x^2 + 1}}{x}$

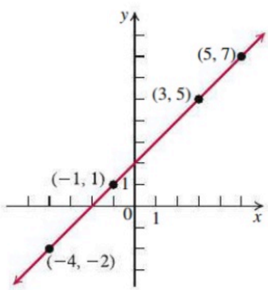
50. $g(x) = \frac{1}{x^2 + 1}$

51. $s(x) = \sqrt{1 - x^2}$

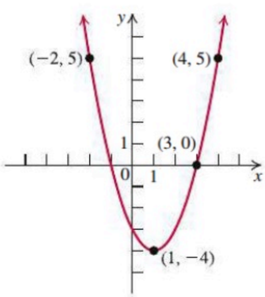
52. $K(x) = \sqrt{x^2 - 4}$

In Exercises 59–62, the graph of a function is given. Find the indicated function values.

59. $f(-4), f(-1), f(3), f(5)$



60. $g(-2), g(1), g(3), g(4)$



In Exercises 71–82, find the average rate of change of the function as *x* changes from *a* to *b*.

71. $f(x) = -2x + 7; a = -1, b = 3$

72. $f(x) = 4x - 9; a = -2, b = 2$

73. $g(x) = 2x^2; a = 0, b = 5$

74. $g(x) = -4x^2; a = -1, b = 4$

75. $h(x) = x^2 - 1; a = -2, b = 0$

76. $h(x) = 2 - x^2; a = 3, b = 4$

77. $f(x) = (3 - x)^2; a = 1, b = 3$

78. $f(x) = (x - 2)^2; a = -1, b = 5$

79. $g(x) = x^3; a = -1, b = 3$

80. $g(x) = -x^3; a = -1, b = 3$

81. $h(x) = \frac{1}{x}; a = 2, b = 6$

82. $h(x) = \frac{4}{x + 3}; a = -2, b = 4$

In Exercises 83–90, find and simplify the difference quotient of the form $\frac{f(x) - f(a)}{x - a}, x \neq a$.

83. $f(x) = 2x, a = 3$

84. $f(x) = 3x + 2, a = 2$

85. $f(x) = -x^2, a = 1$

86. $f(x) = 2x^2, a = -1$

87. $f(x) = 3x^2 + x, a = 2$

88. $f(x) = -2x^2 + x, a = 3$

89. $f(x) = \frac{4}{x}, a = 1$

90. $f(x) = -\frac{4}{x}, a = 1$

SECTION 1.4 Exercises

Concepts and Vocabulary

1. A function f is decreasing if $x_1 < x_2$ implies that ______________ .

2. A function f has a relative maximum at $x = a$ if there is an interval (x_1, x_2) containing a such that ______________ for every x in the interval (x_1, x_2).

3. A function is even if, for all x in the domain of f, we have ______________ .

4. A function that uses different rules for assigning output values on different parts of the domain is called a ______________ .

5. **True or False.** Functions can increase, decrease, or remain constant on different intervals within their domains.

6. **True or False.** The average rate of change of an increasing function is positive.

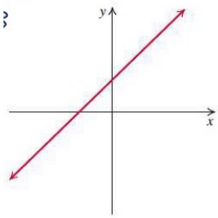
7. **True or False.** At a point on a graph where a function changes direction from increasing to decreasing, the function has a relative minimum.

8. **True or False.** The graph of an odd function is symmetric with respect to the y-axis.

Building Skills

In Exercises 9–16, the graph of a function is given. For each function, determine the intervals over which the function is increasing, decreasing, or constant.

9.



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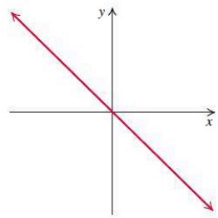

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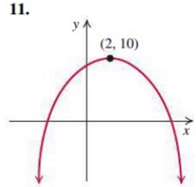

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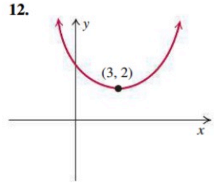

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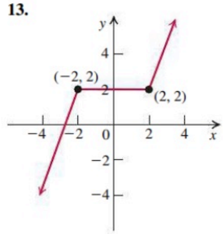

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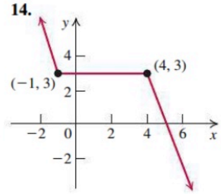

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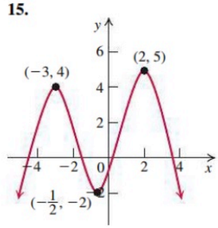

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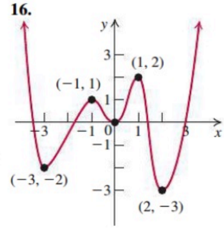


In Exercises 35–48, determine algebraically whether the given function is odd, even, or neither.

35. $f(x) = 2x^4 + 4$

36. $g(x) = 3x^4 - 5$

37. $f(x) = 5x^3 - 3x$

38. $g(x) = 2x^3 + 4x$

39. $f(x) = 2x + 4$

40. $g(x) = 3x + 7$

41. $f(x) = \dfrac{1}{x^2 + 4}$

42. $g(x) = \dfrac{x^2 + 2}{x^4 + 1}$

43. $f(x) = \dfrac{x^3}{x^2 + 1}$

44. $g(x) = \dfrac{x^4 + 3}{2x^3 - 3x}$

45. $f(x) = \dfrac{x}{x^5 - 3x^3}$

46. $g(x) = \dfrac{x^3 + 2x}{2x^5 - 3x}$

SECTION 1.5 Exercises

Concepts and Vocabulary

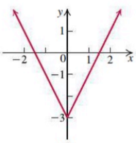
1. The graph of $y = f(x) - 3$ is found by vertically shifting the graph of $y = f(x)$ by three units ______________ .
2. The graph of $y = f(x + 5)$ is found by horizontally shifting the graph of $y = f(x)$ by five units to the ______________ .
3. The graph of $y = f(-x)$ is found by reflecting the graph of $y = f(x)$ about the ______________ .
4. The graph of $y = -f(x)$ is found by reflecting the graph of $y = f(x)$ about the ______________ .
5. **True or False.** The graph of $y = f(bx)$ is a horizontal compression of the graph $y = f(x)$ if $b < 1$.
6. **True or False.** The graphs of $y = f(x)$ and $y = f(x) + 1$ cannot be the same.
7. **True or False.** The graphs of $y = f(x)$ and $y = f(-x)$ cannot be the same.
8. **True or False.** Combining horizontal and vertical shifts preserves the shape of the original graph.

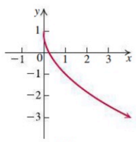
In Exercises 9–22, describe the transformations that produce the graphs of g and h from the graph of f.

9. $f(x) = \sqrt{x}$
   a. $g(x) = \sqrt{x} + 2$
   b. $h(x) = \sqrt{x} - 1$
10. $f(x) = |x|$
    a. $g(x) = |x| + 1$
    b. $h(x) = |x| - 2$
11. $f(x) = x^2$
    a. $g(x) = (x + 1)^2$
    b. $h(x) = (x - 2)^2$
12. $f(x) = \dfrac{1}{x}$
    a. $g(x) = \dfrac{1}{x + 2}$
    b. $h(x) = \dfrac{1}{x - 3}$
13. $f(x) = \sqrt{x}$
    a. $g(x) = \sqrt{x + 1} - 2$
    b. $h(x) = \sqrt{x - 1} + 3$
14. $f(x) = x^2$
    a. $g(x) = -x^2$
    b. $h(x) = (-x)^2$
15. $f(x) = |x|$
    a. $g(x) = -|x|$
    b. $h(x) = |-x|$
16. $f(x) = \sqrt{x}$
    a. $g(x) = 2\sqrt{x}$
    b. $h(x) = \sqrt{2x}$
17. $f(x) = \dfrac{1}{x}$
    a. $g(x) = \dfrac{2}{x}$
    b. $h(x) = \dfrac{1}{2x}$
18. $f(x) = x^3$
    a. $g(x) = (x - 2)^3 + 1$
    b. $h(x) = -(x + 1)^3 + 2$
19. $f(x) = \sqrt{x}$
    a. $g(x) = -\sqrt{x} + 1$
    b. $h(x) = \sqrt{-x} + 1$
20. $f(x) = [\![x]\!]$
    a. $g(x) = [\![x - 1]\!] + 2$
    b. $h(x) = 3[\![x]\!] - 1$
21. $f(x) = \sqrt[3]{x}$
    a. $g(x) = \sqrt[3]{x} + 1$
    b. $h(x) = \sqrt[3]{x + 1}$
22. $f(x) = \sqrt[3]{x}$
    a. $g(x) = 2\sqrt[3]{1 - x} + 4$
    b. $h(x) = -\sqrt[3]{x - 1} + 3$

In Exercises 23–34, match each function with its graph (a)–(l).

23. $y = -|x| + 1$
24. $y = -\sqrt{-x}$
25. $y = \sqrt{x^2}$
26. $y = \frac{1}{2}|x|$
27. $y = \sqrt{x + 1}$
28. $y = 2|x| - 3$
29. $y = 1 - 2\sqrt{x}$
30. $y = -|x - 1| + 1$
31. $y = (x - 1)^2$
32. $y = -x^2 + 3$
33. $y = -2(x - 3)^2 - 1$
34. $y = 3 - \sqrt{1 - x}$


(a)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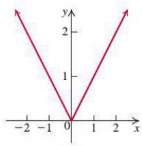
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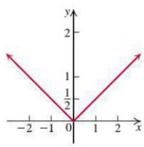
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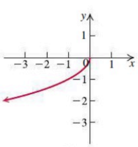
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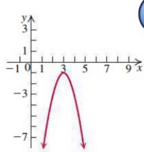
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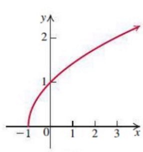
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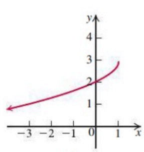
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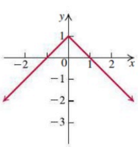
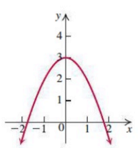
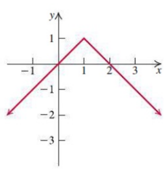

(e)


(f)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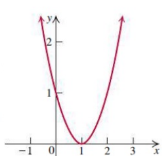

(l)

In Exercises 35–62, graph each function by starting with a function from the library of functions and then using the techniques of shifting, compressing, stretching, and/or reflecting.

35. $f(x) = x^2 - 2$

36. $f(x) = x^2 + 3$

37. $g(x) = \sqrt{x} + 1$

38. $g(x) = \sqrt{x} - 4$

39. $f(x) = |x| + 2$

40. $f(x) = |x| - 1$

41. $f(x) = x^3 + 2$

42. $f(x) = x^3 - 1$

43. $f(x) = \frac{1}{x} + 1$

44. $f(x) = \frac{1}{x} - 2$

45. $f(x) = (x - 3)^3$

46. $f(x) = (x + 2)^3$

47. $f(x) = \sqrt{x - 1}$

48. $f(x) = \sqrt{x + 2}$

49. $h(x) = |x + 1|$

50. $h(x) = |x - 2|$

51. $f(x) = (x + 1)^3$

52. $f(x) = (x - 3)^3$

53. $f(x) = \frac{1}{x - 3}$

54. $f(x) = \frac{1}{x + 2}$

55. $f(x) = \sqrt{-x}$

56. $f(x) = -\sqrt{x}$

57. $f(x) = -x^2$

58. $f(x) = -x^3$

59. $f(x) = 2x^2$

60. $f(x) = \frac{1}{3}x^2$

61. $f(x) = 2|x|$

62. $f(x) = \frac{1}{3}|x|$

In Exercises 63–74, graph each function by starting with a function from the library of functions and then combining shifting and reflecting techniques.

63. $f(x) = (x - 2)^2 + 1$

64. $f(x) = (x - 3)^2 - 5$

65. $f(x) = 5 - (x - 3)^2$

66. $f(x) = 2 - (x + 1)^2$

67. $f(x) = \sqrt{x + 1} - 3$

68. $f(x) = \sqrt{x - 2} + 1$

69. $f(x) = \sqrt{1 - x} + 2$

70. $f(x) = -\sqrt{x + 2} - 3$

71. $f(x) = |x - 1| - 2$

72. $f(x) = -|x + 3| + 1$

73. $f(x) = \frac{1}{x - 1} + 3$

74. $f(x) = 2 - \frac{1}{x + 2}$

In Exercises 75–82, graph each function by starting with a function from the library of functions and then combining shifting, compressing, stretching, and/or reflecting techniques

75. $f(x) = 2(x + 1)^2 - 1$

76. $f(x) = \frac{1}{3}(x + 1)^2 + 2$

77. $f(x) = 2 - \frac{1}{2}(x - 3)^2$

78. $f(x) = 1 - 3(x - 3)^2$

79. $f(x) = 2\sqrt{x + 1} - 3$

80. $f(x) = \sqrt{2x - 2} + 1$

81. $f(x) = -2|x - 1| + 2$

82. $f(x) = -\frac{1}{2}|3 - x| - 1$

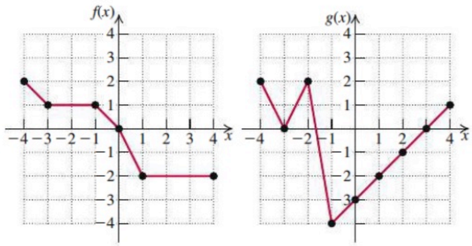
SECTION 1.6 Exercises

Concepts and Vocabulary

1. $(f \cdot g)(x) =$ ____________.
2. The domain of the function $f + g$ consists of those values of x that are ____________ to the domains of f and g.
3. The composition of the function f with the function g is written as $f \circ g$ and is defined by $f \circ g(x) =$ ____________.
4. The domain of the composite function $f \circ g$ consists of those values of x in the domain of g for which $g(x)$ ____________.
5. **True or False.** We always have $f \circ g = g \circ f$.
6. **True or False.** If $f(1) = 2$ and $g(2) = 1$, then $(f \circ g)(2) = 2$.
7. **True or False.** The domain of $f \cdot g$ and the domain of $\frac{f}{g}$ are always the same.
8. **True or False.** A function may be decomposed into simpler functions in several different ways.

Building Skills

In Exercises 9–20, use the graphs of f and g shown in the figure to evaluate each expression.



9. $(f + g)(-2)$
10. $(f + g)(2)$
11. $(f - g)(4)$
12. $(f - g)(-1)$
13. $(f \cdot g)(-1)$
14. $(f \cdot g)(2)$
15. $\left(\frac{f}{g}\right)(-2)$
16. $\left(\frac{f}{g}\right)(2)$
17. $(f \circ g)(1)$
18. $(g \circ f)(1)$
19. $(f \circ g)(-3)$
20. $(g \circ f)(-3)$

In Exercises 21–24, functions f and g are given. Find each of the given values.

a. $(f + g)(-1)$ b. $(f - g)(0)$

c. $(f \cdot g)(2)$ d. $\left(\frac{f}{g}\right)(1)$

21. $f(x) = 2x;\ g(x) = -x$
22. $f(x) = 1 - x^2;\ g(x) = x + 1$
23. $f(x) = \dfrac{1}{\sqrt{x + 2}};\ g(x) = 2x + 1$
24. $f(x) = \dfrac{x}{x^2 - 6x + 8};\ g(x) = 3 - x$

In Exercises 25–38, functions f and g are given. Find each of the following functions and state its domain.

a. $f + g$ b. $f - g$ c. $f \cdot g$

d. $\dfrac{f}{g}$ e. $\dfrac{g}{f}$

25. $f(x) = x - 3;\ g(x) = x^2$
26. $f(x) = 2x - 1;\ g(x) = x^2$
27. $f(x) = x^3 - 1;\ g(x) = 2x^2 + 5$
28. $f(x) = x^2 - 4;\ g(x) = x^2 - 6x + 8$
29. $f(x) = 2x - 1;\ g(x) = \sqrt{x}$
30. $f(x) = x - 1;\ g(x) = \sqrt{x}$
31. $f(x) = x - 6;\ g(x) = \sqrt{x - 3}$
32. $f(x) = x + 2;\ g(x) = \sqrt{1 - x}$
33. $f(x) = 1 - \dfrac{2}{x + 1};\ g(x) = \dfrac{1}{x}$

In Exercises 43 and 44, use each diagram to evaluate $(g \circ f)(x)$. Then evaluate $(g \circ f)(2)$ and $(g \circ f)(-3)$.

43. $x \rightarrow$ [$f(x) = x^2 - 1$] $\rightarrow$ [$g(x) = 2x + 3$] $\rightarrow$

44. $x \rightarrow$ [$f(x) = |x + 1|$] $\rightarrow$ [$g(x) = 3x^2 - 1$] $\rightarrow$

In Exercises 45–56, let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 3$. Evaluate each expression.

45. $(f \circ g)(2)$
46. $(g \circ f)(2)$
47. $(f \circ g)(-3)$
48. $(g \circ f)(-5)$
49. $(f \circ g)(0)$
50. $(g \circ f)\left(\frac{1}{2}\right)$
51. $(f \circ g)(-c)$
52. $(f \circ g)(c)$
53. $(g \circ f)(a)$
54. $(g \circ f)(-a)$
55. $(f \circ f)(1)$
56. $(g \circ g)(-1)$

In Exercises 57–62, the functions f and g are given. Find $f \circ g$ and its domain.

57. $f(x) = \dfrac{2}{x + 1};\ g(x) = \dfrac{1}{x}$
58. $f(x) = \dfrac{1}{x - 1};\ g(x) = \dfrac{2}{x + 3}$
59. $f(x) = \sqrt{x - 3};\ g(x) = 2 - 3x$
60. $f(x) = \dfrac{x}{x - 1};\ g(x) = 2 + 5x$
61. $f(x) = |x|;\ g(x) = x^2 - 1$
62. $f(x) = 3x - 2;\ g(x) = |x - 1|$

In Exercises 63–78, the functions f and g are given. Find each composite function and describe its domain.

a. $f \circ g$ **b.** $g \circ f$

c. $f \circ f$ **d.** $g \circ g$

63. $f(x) = 2x - 3;\ g(x) = x + 4$

64. $f(x) = x - 3;\ g(x) = 3x - 5$

65. $f(x) = 1 - 2x;\ g(x) = 1 + x^2$

66. $f(x) = 2x - 3;\ g(x) = 2x^2$

67. $f(x) = 2x^2 + 3x;\ g(x) = 2x - 1$

68. $f(x) = x^2 + 3x;\ g(x) = 2x$

69. $f(x) = x^2;\ g(x) = \sqrt{x}$

70. $f(x) = x^2 + 2x;\ g(x) = \sqrt{x + 2}$

71. $f(x) = \dfrac{1}{2x - 1};\ g(x) = \dfrac{1}{x^2}$

72. $f(x) = x - 1;\ g(x) = \dfrac{x}{x + 1}$

73. $f(x) = \sqrt{x - 1};\ g(x) = \sqrt{4 - x}$

74. $f(x) = x^2 - 4;\ g(x) = \sqrt{4 - x^2}$

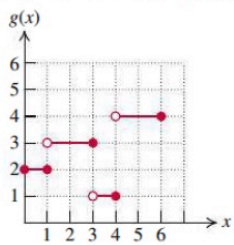
75. $f(x) = \dfrac{1 - x}{x + 2};\ g(x) = \dfrac{x + 3}{x - 4}$

76. $f(x) = \dfrac{x + 2}{x - 3};\ g(x) = \dfrac{x + 1}{x - 1}$

77. $f(x) = 1 + \dfrac{1}{x};\ g(x) = \dfrac{1 + x}{1 - x}$

78. $f(x) = \sqrt[3]{x + 1};\ g(x) = x^3 + 1$

In Exercises 79–82, let $g(x)$ be a piecewise function given below. For each f, find the domain of the composite function $f \circ g$.



79. $f(x) = \dfrac{3}{x - 3}$

80. $f(x) = \dfrac{2}{2 - x}$

81. $f(x) = \sqrt{3 - x}$

82. $f(x) = \sqrt{x - 2}$

In Exercises 87–96, express the given function H as a composition of two functions f and g such that $H(x) = (f \circ g)(x)$.

87. $H(x) = \sqrt{x + 2}$

88. $H(x) = |3x + 2|$

89. $H(x) = (x^2 - 3)^{10}$

90. $H(x) = \sqrt{3x^2 + 5}$

91. $H(x) = \dfrac{1}{3x - 5}$

92. $H(x) = \dfrac{5}{2x + 3}$

93. $H(x) = \sqrt[3]{x^2 - 7}$

94. $H(x) = \sqrt[4]{x^2 + x + 1}$

95. $H(x) = \dfrac{1}{|x^3 - 1|}$

96. $H(x) = \sqrt[3]{1 + \sqrt{x}}$

In Exercises 97–102, the functions f and g are given. Find the average rate of change of the composite function $f \circ g$ as x changes from a to b.

97. $f(x) = x^2 + 2;\ g(x) = 1 - 2x;\ a = 1, b = 2$

98. $f(x) = 1 - x^2;\ g(x) = 1 + 3x;\ a = -1, b = 1$

99. $f(x) = x^3 + 2;\ g(x) = 1 - x^2;\ a = 1, b = 2$

100. $f(x) = 1 - x^3;\ g(x) = x^2 + 1;\ a = -1, b = 0$

101. $f(x) = \dfrac{1}{4 + x};\ g(x) = x^2 - 1;\ a = 1, b = 2$

102. $f(x) = \dfrac{1}{2 + x};\ g(x) = x^2 + 1;\quad a = 0, b = 2$

SECTION 1.7 Exercises

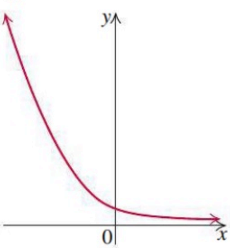
Concepts and Vocabulary

1. A function f is one-to-one if for any two different numbers x_1 and x_2, $x_1 \neq x_2$, in the domain of f we have ______________.
2. A function f is one-to-one if every horizontal line intersects the graph of f at no more than ______________.
3. $f^{-1} \circ f(x) =$ ______________.
4. The graph of f^{-1} is a reflection of the graph of f about the line ______________.
5. **True or False.** If a function f is one-to-one, then it has inverse f^{-1}.
6. **True or False.** $f^{-1}(x) = \dfrac{1}{f(x)}$.
7. **True or False.** The domain of f^{-1} equals the domain of f.
8. **True or False.** If a point (a, b) is on the graph of a one-to one function f, then the point (b, a) is on the graph of f^{-1}.

Building Skills

In Exercises 9–16, the graph of a function is given. Use the horizontal-line test to determine whether the function is one-to-one.

9.



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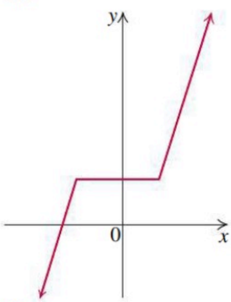

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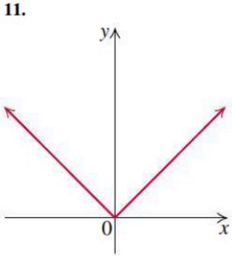

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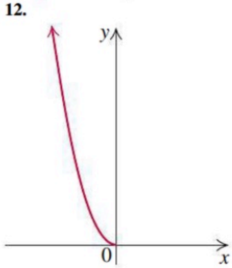

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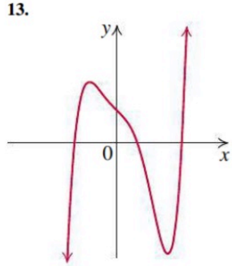

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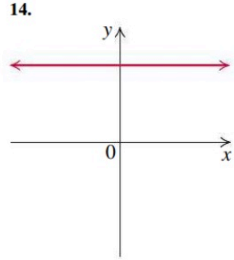

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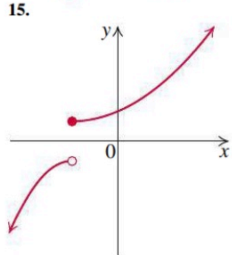

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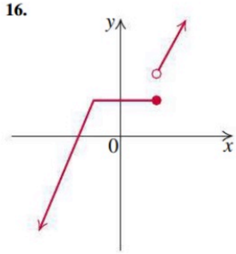

In Exercises 17–24, assume that the function f is one-to-one with domain: $(-\infty, \infty)$.

17. If $f(2) = 7$, find $f^{-1}(7)$.

18. If $f^{-1}(4) = -7$, find $f(-7)$.

19. If $f(-1) = 2$, find $f^{-1}(2)$.

20. If $f^{-1}(-3) = 5$, find $f(5)$.

21. For $f(x) = 2x - 3$, find each of the following.
a. $f(3)$ **b.** $f^{-1}(3)$
c. $(f \circ f^{-1})(19)$ **d.** $(f \circ f^{-1})(5)$

22. For $f(x) = x^3$, find each of the following.
a. $f(2)$ **b.** $f^{-1}(8)$
c. $(f \circ f^{-1})(15)$ **d.** $(f^{-1} \circ f)(27)$

23. For $f(x) = x^3 + 1$, find each of the following.
a. $f(1)$
b. $f^{-1}(2)$
c. $(f \circ f^{-1})(269)$

24. For $g(x) = \sqrt[3]{2x^3 - 1}$, find each of the following.
a. $g(1)$
b. $g^{-1}(1)$
c. $(g^{-1} \circ g)(135)$

25. Determine which of the functions described below are one-to-one. The function that assigns to each shirt in a store its
a. color.
b. price.
c. bar code.

26. Determine which of the functions described below are one-to-one. The function that assigns to each person in the United States their
a. Social Security number.
b. first name.
c. last name.

In Exercises 27–30, determine the order and write down the steps to undo the following sequence of actions.

27. First: Open the fridge door. Second: Take the milk out of the fridge.

28. First: Put socks on. Second: Put shoes on.

29. First: Wake up. Second: Put makeup on.

30. First: Dig a hole. Second: Plant a tree.

In Exercises 31–34 determine the order and steps to undo the following sequence of actions. Write the original function and the inverse function associated with the final result of these actions.

31. First: Multiply by 2. Second: Add 3.

32. First: Subtract 2. Second: Multiply by 3.

33. First: Cube it. Second: Add 2.

34. First: Subtract 3. Second: Cube it.

In Exercises 35–42, show that f and g are inverses of each other by verifying that $f(g(x)) = x = g(f(x))$.

35. $f(x) = 3x + 1$; $g(x) = \dfrac{x - 1}{3}$

36. $f(x) = 2 - 3x$; $g(x) = \dfrac{2 - x}{3}$

37. $f(x) = x^3$; $g(x) = \sqrt[3]{x}$

38. $f(x) = \dfrac{1}{x}$; $g(x) = \dfrac{1}{x}$

39. $f(x) = 2x^5 + 1$; $g(x) = \sqrt[5]{\dfrac{x - 1}{2}}$

40. $f(x) = (1 - 3x)^3$; $g(x) = \dfrac{1 - \sqrt[3]{x}}{3}$

41. $f(x) = \dfrac{x - 1}{x + 2}$; $g(x) = \dfrac{1 + 2x}{1 - x}$

42. $f(x) = \dfrac{3x + 2}{x - 1}$; $g(x) = \dfrac{x + 2}{x - 3}$

In Exercises 51–60, the function f is one-to-one. Find the f^{-1} and verify your answer. Also find the domain and range of the given function f.

51. $f(x) = 3x - 1$ **52.** $f(x) = 2x + 3$

53. $f(x) = \sqrt[3]{\dfrac{x + 1}{3}} + 2$ **54.** $f(x) = \sqrt[3]{\dfrac{x - 2}{3}} - 1$

55. $f(x) = (3x - 1)^3 + 2$ **56.** $f(x) = (2x + 1)^3 - 3$

57. $f(x) = \dfrac{2}{1 + x}$ **58.** $f(x) = 1 - \dfrac{1}{x + 1}$

59. $f(x) = \dfrac{x + 1}{x - 2}$ **60.** $f(x) = \dfrac{1 - 2x}{1 + x}$

In Exercises 61–64, sketch the graph of f using appropriate transformations and confirm that f is one-to-one by using the horizontal line test. Find f^{-1}. Find the domain and range of f and f^{-1}.

61. $f(x) = x^2 + 1, x \geq 0$ **62.** $f(x) = x^2 - 4, x \leq 0$

63. $f(x) = -x^2 + 2, x \leq 0$ **64.** $f(x) = -x^2 - 1, x \geq 0$

In Exercises 65–76, sketch the graph of the function and its inverse on the same coordinate axes.

65. $f(x) = 15 - 3x$ **66.** $g(x) = 2x + 5$

67. $f(x) = \sqrt{4 - x^2}, x \geq 0$ **68.** $f(x) = -\sqrt{9 - x^2}, x \geq 0$

69. $f(x) = \sqrt{x} + 3$ **70.** $f(x) = 4 - \sqrt{x}$

71. $g(x) = \sqrt[3]{x + 1}$ **72.** $h(x) = \sqrt[3]{1 - x}$

73. $f(x) = \dfrac{1}{x - 1}, x \neq 1$ **74.** $g(x) = 1 - \dfrac{1}{x}, x \neq 0$

75. $f(x) = 2 + \sqrt{x + 1}$ **76.** $f(x) = -1 + \sqrt{x + 2}$

In Exercises 77–80, assume that the given function is one-to-one. Find the inverse of the function. Also find the domain and the range of the given function.

77. $f(x) = \dfrac{x + 1}{x - 2}, x \neq 2$ **78.** $g(x) = \dfrac{x + 2}{x + 1}, x \neq -1$

79. $f(x) = \dfrac{1 - 2x}{1 + x}, x \neq -1$ **80.** $h(x) = \dfrac{x - 1}{x - 3}, x \neq 3$

SECTION 7.1 Exercises

Concepts and Vocabulary

1. The ordered pair (a, b) is a(n) ________ of a system of equations in x and y providing that when x is replaced with a and y is replaced with b, the resulting equations are true.
2. The two nongraphical methods for solving a system of equations are the ________ and ________ methods.
3. If in the process of solving a system of equations you get an equation of the form $0 = k$, where k is not zero, then the system is ________.
4. If in the process of solving a system of equations you get an equation of the form $0 = 0$, then the system has ________.
5. **True or False.** A system consisting of two identical equations has no solution.
6. **True or False.** If in the process of solving a system of two equations in x and y you get the equation $5 = 5$, then the system has exactly one solution.
7. **True or False.** If $x = 4$, $y = -7$ is the solution of a system of two equations in x and y, then the lines determined by the two equations intersect at the point $(4, -7)$.
8. **True or False.** When solving $\begin{cases} 3x - 2y = 14 \\ 9x + 8y = 23 \end{cases}$ by the addition method, we can eliminate x by multiplying the first equation by -3 and adding the equations.

Building Skills

In Exercises 9–14, determine which ordered pairs are solutions of each system of equations.

9. $\begin{cases} 2x + 3y = 3 \\ 3x - 4y = 13 \end{cases}$ $(1, -3), (3, -1), (6, 3), \left(5, \frac{1}{2}\right)$
10. $\begin{cases} x + 2y = 6 \\ 3x + 6y = 18 \end{cases}$ $(2, 2), (-2, 4), (0, 3), (1, 2)$
11. $\begin{cases} 5x - 2y = 7 \\ -10x + 4y = 11 \end{cases}$ $\left(\frac{5}{4}, 1\right), \left(0, \frac{11}{4}\right), (1, -1), (3, 4)$
12. $\begin{cases} x - 2y = -5 \\ 3x - y = 5 \end{cases}$ $(1, 3), (-5, 0), (3, 4), (3, -4)$
13. $\begin{cases} x + y = 1 \\ \frac{1}{2}x + \frac{1}{3}y = 2 \end{cases}$ $(0, 1), (1, 0), \left(\frac{2}{3}, \frac{3}{2}\right), (10, -9)$
14. $\begin{cases} \frac{2}{x} + \frac{3}{y} = 2 \\ \frac{6}{x} + \frac{18}{y} = 9 \end{cases}$ $(3, 2), (2, 3), (4, 3), (3, 4)$

In Exercises 15–24, estimate the solution(s) (if any) of each system by the graphical method. Check your solution(s). For any dependent equations, write your answer with x being arbitrary.

15. $\begin{cases} x + y = 3 \\ x - y = 1 \end{cases}$
16. $\begin{cases} x + y = 10 \\ x - y = 2 \end{cases}$
17. $\begin{cases} x + 2y = 6 \\ 2x + y = 6 \end{cases}$
18. $\begin{cases} 2x - y = 4 \\ x - y = 3 \end{cases}$
19. $\begin{cases} 3x - y = -9 \\ y = 3x + 6 \end{cases}$
20. $\begin{cases} 5x + 2y = 10 \\ y = -\frac{5}{2}x - 5 \end{cases}$
21. $\begin{cases} x + y = 7 \\ y = 2x \end{cases}$
22. $\begin{cases} y - x = 2 \\ y + x = 9 \end{cases}$
23. $\begin{cases} 3x + y = 12 \\ y = -3x + 12 \end{cases}$
24. $\begin{cases} 2x + 3y = 6 \\ 6y = -4x + 12 \end{cases}$

In Exercises 25–38, determine whether each system is *consistent* or *inconsistent*. If the system is consistent, determine whether the equations are dependent or independent. Do not solve the system.

25. $\begin{cases} y = -2x + 3 \\ y = 3x + 5 \end{cases}$
26. $\begin{cases} 3x + y = 5 \\ 2x + y = 4 \end{cases}$
27. $\begin{cases} 2x + 3y = 5 \\ 3x + 2y = 7 \end{cases}$
28. $\begin{cases} 2x - 4y = 5 \\ 3x + 5y = -6 \end{cases}$
29. $\begin{cases} 3x + 5y = 7 \\ 6x + 10y = 14 \end{cases}$
30. $\begin{cases} 3x - y = 2 \\ 9x - 3y = 6 \end{cases}$
31. $\begin{cases} x + 2y = -5 \\ 2x - y = 4 \end{cases}$
32. $\begin{cases} x + 2y = -2 \\ 2x - 3y = 5 \end{cases}$
33. $\begin{cases} 2x - 3y = 5 \\ 6x - 9y = 10 \end{cases}$
34. $\begin{cases} 3x + y = 2 \\ 15x + 5y = 15 \end{cases}$
35. $\begin{cases} -3x + 4y = 5 \\ \frac{9}{2}x - 6y = \frac{15}{2} \end{cases}$
36. $\begin{cases} 6x + 5y = 11 \\ 9x + \frac{15}{2}y = 21 \end{cases}$
37. $\begin{cases} 7x - 2y = 3 \\ 11x - \frac{3}{2}y = 8 \end{cases}$
38. $\begin{cases} 4x + 7y = 10 \\ 10x + \frac{35}{2}y = 25 \end{cases}$

In Exercises 39–48, solve each system of equations by the substitution method. Check your solutions. For any dependent equations, write your answer in the ordered pair form given in Example 5.

39. $\begin{cases} y = 2x + 1 \\ 5x + 2y = 9 \end{cases}$

40. $\begin{cases} x = 3y - 1 \\ 2x - 3y = 7 \end{cases}$

41. $\begin{cases} 3x - y = 5 \\ x + y = 7 \end{cases}$

42. $\begin{cases} 2x + y = 2 \\ 3x - y = -7 \end{cases}$

43. $\begin{cases} 2x - y = 5 \\ -4x + 2y = 7 \end{cases}$

44. $\begin{cases} 3x + 2y = 5 \\ -9x - 6y = 15 \end{cases}$

45. $\begin{cases} x - y = 2 \\ x^2 - 4x + y^2 = -2 \end{cases}$

46. $\begin{cases} 2x - y = 1 \\ x^2 - 8y + y^2 = -6 \end{cases}$

47. $\begin{cases} x - 2y = 5 \\ -3x + 6y = -15 \end{cases}$

48. $\begin{cases} x + y = 3 \\ 2x + 2y = 6 \end{cases}$

In Exercises 49–58, solve each system of equations by the elimination method. Check your solutions. For any dependent equations, write your answer as in Example 5.

49. $\begin{cases} x - y = 1 \\ x + y = 5 \end{cases}$

50. $\begin{cases} 2x - 3y = 5 \\ 3x + 2y = 14 \end{cases}$

51. $\begin{cases} x + y = 0 \\ 2x + 3y = 3 \end{cases}$

52. $\begin{cases} x + y = 3 \\ 3x + y = 1 \end{cases}$

53. $\begin{cases} x^2 + 2y^2 = 12 \\ -5x^2 + 7y^2 = 8 \end{cases}$

54. $\begin{cases} x^2 - 6y^2 = 19 \\ 3x^2 + 2y^2 = 77 \end{cases}$

55. $\begin{cases} x - y = 2 \\ -2x + 2y = 5 \end{cases}$

56. $\begin{cases} x + y = 5 \\ 2x + 2y = -10 \end{cases}$

57. $\begin{cases} 4x + 6y = 12 \\ 2x + 3y = 6 \end{cases}$

58. $\begin{cases} 4x + 7y = -3 \\ -8x - 14y = 6 \end{cases}$

In Exercises 59–78, use any method to solve each system of equations. For any dependent equations, write your answer as in Example 5.

59. $\begin{cases} 2x + y = 9 \\ 2x - 3y = 5 \end{cases}$

60. $\begin{cases} x + 2y = 10 \\ x - 2y = -6 \end{cases}$

61. $\begin{cases} 2x + 5y = 2 \\ x + 3y = 2 \end{cases}$

62. $\begin{cases} 4x - y = 6 \\ 3x - 4y = 11 \end{cases}$

63. $\begin{cases} 2x + 3y = 7 \\ 3x + y = 7 \end{cases}$

64. $\begin{cases} x = 3y + 4 \\ x = 5y + 10 \end{cases}$

65. $\begin{cases} 2x + 3y = 9 \\ 3x + 2y = 11 \end{cases}$

66. $\begin{cases} 3x - 4y = 0 \\ y = \dfrac{2x + 1}{3} \end{cases}$

67. $\begin{cases} \dfrac{x}{4} + \dfrac{y}{6} = 1 \\ x + 2(x - y) = 7 \end{cases}$

68. $\begin{cases} \dfrac{x}{3} + \dfrac{y}{5} = 12 \\ x - y = 4 \end{cases}$

69. $\begin{cases} x - y^2 = 2 \\ 2x + 3y = 3 \end{cases}$

70. $\begin{cases} x + 2y = 6 \\ y - x^2 = 0 \end{cases}$

71. $\begin{cases} 5x - 2y = 7 \\ x^2 + y^2 = 2 \end{cases}$

72. $\begin{cases} x - 2y = -5 \\ x^2 + y^2 = 25 \end{cases}$

73. $\begin{cases} x^2 + y^2 = 20 \\ x^2 - y^2 = 12 \end{cases}$

74. $\begin{cases} x^2 - y^2 = 9 \\ 4x^2 + 5y^2 = 180 \end{cases}$