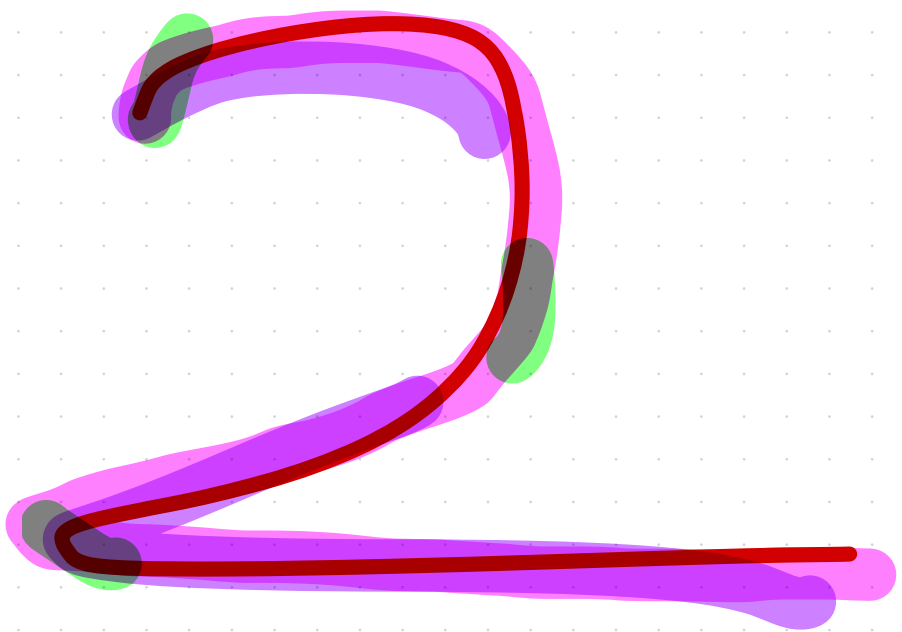
- - - - - - - - - - - - - - - -

UNIT 2 - Quadratic, exponential, and logarithmic functions

2.1 quadratic functions, graphing
2.2 polynomial functions, end behavior, finding zeros
2.3 polynomial long division, factor theorem, rational root test
2.4 rational functions, vertical and horizontal asymptotes, graphing
2.5 solving polynomial inequalities, sign charts

3.1 exponential functions, graphing, compound interest, e^x
3.2 logarithmic functions, exponential vs. logarithmic form, evaluating logs, graphing
3.3 properties of logs, growth and decay, half-life
3.4 solving exponential and logarithmic equations



Building Skills

In Exercises 9–16, match each quadratic function with its graph.

9. $y = -\frac{1}{3}x^2$

10. $y = -3x^2$

11. $y = -3(x + 1)^2$

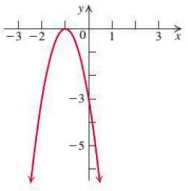
12. $y = 2(x + 1)^2$

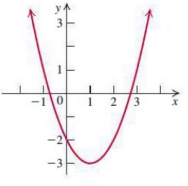
13. $y = (x - 1)^2 + 2$

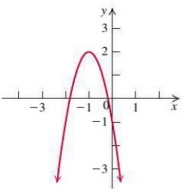
14. $y = (x - 1)^2 - 3$

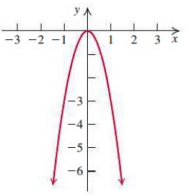
15. $y = 2(x + 1)^2 - 3$

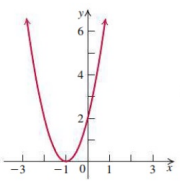
16. $y = -3(x + 1)^2 + 2$

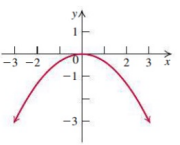

(a)

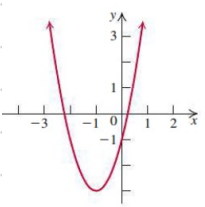

(b)

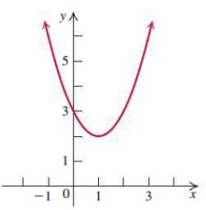

(c)


(d)


(e)


(f)


(g)


(h)

In Exercises 17–20, find a quadratic function of the form $y = ax^2$ that passes through the given point.

17. $(2, -8)$

18. $(-3, 3)$

19. $(2, 20)$

20. $(-3, -6)$

In Exercises 21–30, find the quadratic function $y = f(x)$ that has the given vertex and whose graph passes through the given point. Write the function in standard form.

21. Vertex $(0, 0)$; passing through $(-2, 8)$

22. Vertex $(2, 0)$; passing through $(1, 3)$

23. Vertex $(-3, 0)$; passing through $(-5, -4)$

24. Vertex $(0, 1)$; passing through $(-1, 0)$

25. Vertex $(2, 5)$; passing through $(3, 7)$

26. Vertex $(-3, 4)$; passing through $(0, 0)$

27. Vertex $(2, -3)$; passing through $(-5, 8)$

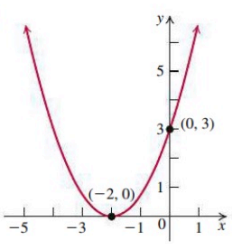
28. Vertex $(-3, -2)$; passing through $(0, -8)$

29. Vertex $\left(\frac{1}{2}, \frac{1}{2}\right)$; passing through $\left(\frac{3}{4}, -\frac{1}{4}\right)$

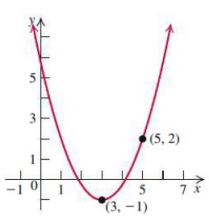
30. Vertex $\left(-\frac{3}{2}, -\frac{5}{2}\right)$; passing through $\left(1, \frac{55}{8}\right)$

In Exercises 31–34, the graph of a quadratic function $y = f(x)$ is given. Find the standard form of the function.

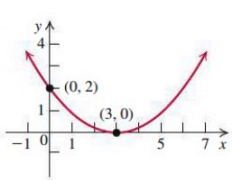
31.



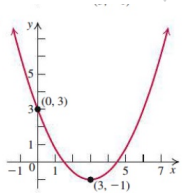
33.



32.



34.



In Exercises 35–44, graph each function by starting with the graph of $y = x^2$ and using transformations.

35. $f(x) = 3x^2$

36. $f(x) = \frac{1}{3}x^2$

37. $g(x) = (x - 4)^2$

38. $g(x) = (x + 3)^2$

39. $f(x) = -2x^2 - 4$

40. $f(x) = -x^2 + 3$

41. $g(x) = (x - 3)^2 + 2$

42. $g(x) = (x + 1)^2 - 3$

43. $f(x) = -3(x - 2)^2 + 4$

44. $f(x) = -2(x + 1)^2 + 3$

In Exercises 45–52, graph the given function by writing it in the standard form $y = a(x - h)^2 + k$ and then using transformations on $y = x^2$. Find the vertex, the axis of symmetry, and the x- and y-intercepts.

45. $y = x^2 + 4x$

46. $y = x^2 - 2x + 2$

47. $y = 6x - 10 - x^2$

48. $y = 8 + 3x - x^2$

49. $y = 2x^2 - 8x + 9$

50. $y = 3x^2 + 12x - 7$

51. $y = -3x^2 + 18x - 11$

52. $y = -5x^2 - 20x + 13$

In Exercises 53–60, (a) determine whether the graph of the given quadratic function opens up or down, (b) find the vertex $(h, k) = \left(-\frac{b}{2a}, f\left(-\frac{b}{2a}\right)\right)$, (c) find the axis of symmetry, (d) find the x- and y-intercepts, and (e) sketch the graph of the function.

53. $y = x^2 - 8x + 15$

54. $y = x^2 + 8x + 13$

55. $y = x^2 - x - 6$

56. $y = x^2 + x - 2$

57. $y = x^2 - 2x + 4$

58. $y = x^2 - 4x + 5$

59. $y = 6 - 2x - x^2$

60. $y = 2 + 5x - 3x^2$

In Exercises 61–68, a quadratic function f is given. (a) Determine whether the given quadratic function has a maximum value or a minimum value. Then find this value. (b) Find the range of f.

61. $f(x) = x^2 - 4x + 3$

62. $f(x) = -x^2 + 6x - 8$

63. $f(x) = -4 + 4x - x^2$

64. $f(x) = x^2 - 6x + 9$

65. $f(x) = 2x^2 - 8x + 3$

66. $f(x) = 3x^2 + 12x - 5$

67. $f(x) = -4x^2 + 12x + 7$

68. $f(x) = 8x - 5 - 2x^2$

Building Skills

In Exercises 9–14, for each polynomial function, find the degree, the leading term, and the leading coefficient.

9. $f(x) = 2x^5 - 5x^2$

10. $f(x) = 3 - 5x - 7x^4$

11. $f(x) = \dfrac{2x^3 + 7x}{3}$

12. $f(x) = -7x + 11 + \sqrt{2}x^3$

13. $f(x) = \pi x^4 + 1 - x^2$

14. $f(x) = 5$

In Exercises 35–42, describe the end behavior of the polynomial function *f*.

35. $f(x) = x - x^3$

36. $f(x) = 2x^3 - 2x^2 + 1$

37. $f(x) = 4x^4 + 2x^3 + 1$

38. $f(x) = -x^4 + 3x^3 + x$

39. $f(x) = (x + 2)^2(2x - 1)$

40. $f(x) = (x - 2)^3(2x + 1)$

41. $f(x) = (x + 2)^2(4 - x)$

42. $f(x) = (x + 3)^3(2 - x)$

In Exercises 43–50, Find the real zeros of *f* and state the multiplicity for each zero. State whether the graph of *f* crosses or touches the *x*-axis at each zero.

43. $f(x) = 3(x - 1)(x + 2)(x - 3)$

44. $f(x) = -5(x + 1)(x + 2)(x - 3)$

45. $f(x) = (x + 2)^2(2x - 1)$

46. $f(x) = (x - 2)^3(2x + 1)$

47. $f(x) = x^2(x^2 - 9)(3x + 2)^3$

48. $f(x) = -x^3(x^2 - 4)(3x - 2)^2$

49. $f(x) = (x^2 + 1)(3x - 2)^2$

50. $f(x) = (x^2 + 1)(x + 1)(x - 2)$

In Exercises 63–74, sketch the graph of the polynomial function *f* using the techniques described in this section.

63. $f(x) = 2(x + 1)(x - 2)(x + 4)$

64. $f(x) = -(x - 1)(x + 3)(x - 4)$

65. $f(x) = (x - 1)^2(x + 3)(x - 4)$

66. $f(x) = -x^2(x + 1)(x - 2)$

67. $f(x) = -x^2(x - 3)^2$

68. $f(x) = (x - 2)^2(x + 3)^2$

69. $f(x) = (x - 1)^2(x + 2)^3(x - 3)$

Building Skills

In Exercises 9–16, use long division to find the quotient and the remainder.

9. $\dfrac{6x^2 - x - 2}{2x + 1}$

10. $\dfrac{4x^3 - 2x^2 + x - 3}{2x - 3}$

11. $\dfrac{3x^4 - 6x^2 + 3x - 7}{x + 1}$

12. $\dfrac{x^6 + 5x^3 + 7x + 3}{x^2 + 2}$

13. $\dfrac{4x^3 - 4x^2 - 9x + 5}{2x^2 - x - 5}$

14. $\dfrac{y^5 + 3y^4 - 6y^2 + 2y - 7}{y^2 + 2y - 3}$

15. $\dfrac{z^4 - 2z^2 + 1}{z^2 - 2z + 1}$

16. $\dfrac{6x^4 + 13x - 11x^3 - 10 - x^2}{3x^2 - 5 - x}$

In Exercises 47–50, find the set of possible rational zeros of the given function.

47. $f(x) = 3x^3 - 4x^2 + 5$

48. $g(x) = 2x^4 - 5x^2 - 2x + 1$

49. $h(x) = 4x^4 - 9x^2 + x + 6$

50. $F(x) = 6x^6 + 5x^5 + x - 35$

In Exercises 51–66, find all rational zeros of the given polynomial function.

51. $f(x) = x^3 - x^2 - 4x + 4$

52. $f(x) = x^3 + x^2 + 2x + 2$

53. $f(x) = x^3 - 4x^2 + x + 6$

54. $f(x) = x^3 + 3x^2 + 2x + 6$

55. $g(x) = 2x^3 + x^2 - 13x + 6$

56. $g(x) = 3x^3 - 2x^2 + 3x - 2$

57. $g(x) = 6x^3 + 13x^2 + x - 2$

58. $g(x) = 2x^3 + 3x^2 + 4x + 6$

59. $h(x) = 3x^3 + 7x^2 + 8x + 2$

60. $h(x) = 2x^3 + x^2 + 8x + 4$

61. $h(x) = x^4 - x^3 - x^2 - x - 2$

SECTION 2.4 Exercises

Building Skills

In Exercises 9–16, find the domain of each rational function.

9. $f(x) = \dfrac{x - 3}{x + 4}$

10. $f(x) = \dfrac{x + 1}{x - 1}$

11. $g(x) = \dfrac{x - 1}{x^2 + 1}$

12. $g(x) = \dfrac{x + 2}{x^2 + 4}$

13. $h(x) = \dfrac{x - 3}{x^2 - x - 6}$

14. $h(x) = \dfrac{x - 7}{x^2 - 6x - 7}$

15. $F(x) = \dfrac{2x + 3}{x^2 - 6x + 8}$

16. $F(x) = \dfrac{3x - 2}{x^2 - 3x + 2}$

In Exercises 35–44, find the vertical asymptotes, if any, of the graph of each rational function.

35. $f(x) = \dfrac{x}{x - 1}$

36. $f(x) = \dfrac{x + 3}{x - 2}$

37. $g(x) = \dfrac{(x + 1)(2x - 2)}{(x - 3)(x + 4)}$

38. $g(x) = \dfrac{(2x - 1)(x + 2)}{(2x + 3)(3x - 4)}$

39. $h(x) = \dfrac{x^2 - 1}{x^2 + x - 6}$

40. $h(x) = \dfrac{x^2 - 4}{3x^2 + x - 4}$

41. $f(x) = \dfrac{x^2 - 6x + 8}{x^2 - x - 12}$

42. $f(x) = \dfrac{x^2 - 9}{x^3 - 4x}$

43. $g(x) = \dfrac{2x + 1}{x^2 + x + 1}$

44. $g(x) = \dfrac{x^2 - 36}{x^2 + 5x + 9}$

In Exercises 45–52, find the horizontal asymptote, if any, of the graph of each rational function.

45. $f(x) = \dfrac{x + 1}{x^2 + 5}$

46. $f(x) = \dfrac{2x - 1}{x^2 - 4}$

47. $g(x) = \dfrac{2x - 3}{3x + 5}$

48. $g(x) = \dfrac{3x + 4}{-4x + 5}$

49. $h(x) = \dfrac{x^2 - 49}{x + 7}$

50. $h(x) = \dfrac{x + 3}{x^2 - 9}$

51. $f(x) = \dfrac{2x^2 - 3x + 7}{3x^3 + 5x + 11}$

52. $f(x) = \dfrac{3x^3 + 2}{x^2 + 5x + 11}$

In Exercises 53–58, match the rational function with its graph.

53. $f(x) = \dfrac{2}{x - 3}$

54. $f(x) = \dfrac{x - 2}{x + 3}$

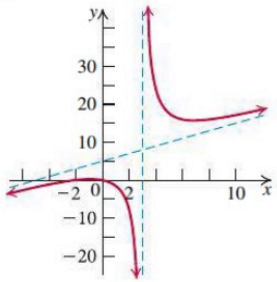
55. $f(x) = \dfrac{1}{x^2 - 2x}$

56. $f(x) = \dfrac{x}{x^2 + 1}$

57. $f(x) = \dfrac{x^2 + 2x}{x - 3}$

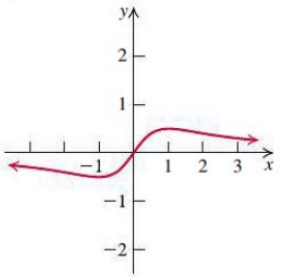
58. $f(x) = \dfrac{x^2}{x^2 - 4}$

a.



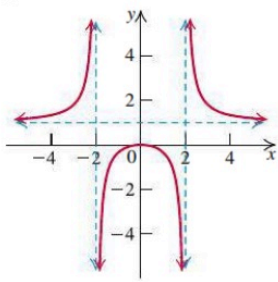
(a)

b.



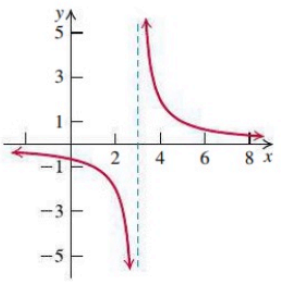
(b)

c.



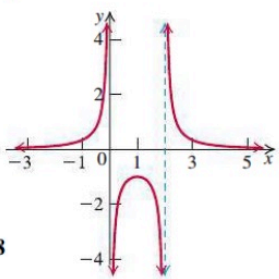
(c)

d.



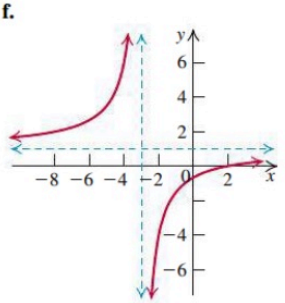
(d)

e.



(e)

f.



(f)

In Exercises 59–80, use the six-step procedure on pages 196–198 to graph each rational function.

59. $f(x) = \dfrac{2x}{x - 3}$

60. $f(x) = \dfrac{-x}{x - 1}$

61. $f(x) = \dfrac{x}{x^2 - 4}$

62. $f(x) = \dfrac{x}{1 - x^2}$

63. $h(x) = \dfrac{-2x^2}{x^2 - 9}$

64. $h(x) = \dfrac{4 - x^2}{x^2}$

65. $f(x) = \dfrac{2}{x^2 - 2}$

66. $f(x) = \dfrac{-2}{x^2 - 3}$

67. $g(x) = \dfrac{x + 1}{(x - 2)(x + 3)}$

68. $g(x) = \dfrac{x - 1}{(x + 1)(x - 2)}$

In Exercises 23–30, sketch the graph of the given function by making a table of values. (Use a calculator if necessary.)

23. $f(x) = 4^x$

24. $g(x) = 10^x$

25. $g(x) = \left(\frac{3}{2}\right)^{-x}$

26. $h(x) = 7^{-x}$

27. $h(x) = \left(\frac{1}{4}\right)^{x}$

28. $f(x) = \left(\frac{1}{10}\right)^{x}$

29. $f(x) = (1.3)^{-x}$

30. $g(x) = (0.7)^{-x}$

31. How are the graphs in Exercises 23 and 27 related? Can we obtain the graph of Exercise 27 from that of Exercise 23? If so, how?

32. Repeat Exercise 31 for the graphs in Exercises 24 and 28.

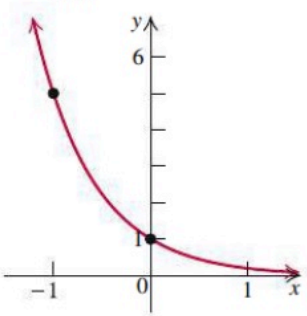
Match each exponential function given in Exercises 33–36 with one of the graphs labeled (a), (b), (c), and (d).

33. $f(x) = 5^x$

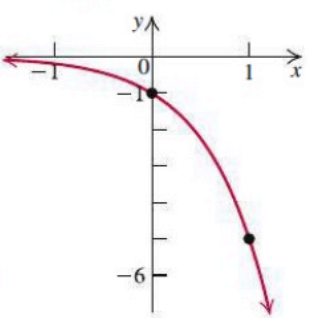
34. $f(x) = -5^x$

35. $f(x) = 5^{-x}$

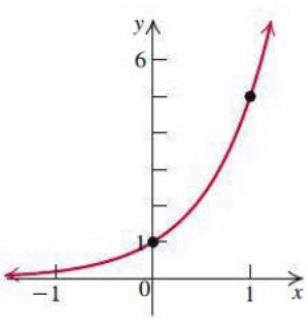
36. $f(x) = 5^{-x} + 1$



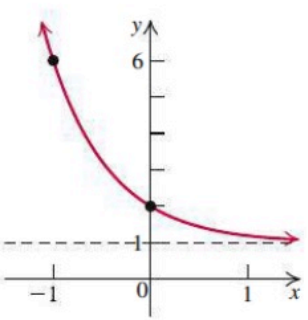
(a)



(b)



(c)



(d)

In Exercises 37–46, start with the graph of the appropriate basic exponential function f and use transformations to sketch the graph of the function g. State the domain and range of g and the horizontal asymptote of its graph.

37. $g(x) = 3^{x-1}$

38. $g(x) = 3^x - 1$

39. $g(x) = 4^{-x}$

40. $g(x) = -4^x$

41. $g(x) = -2.5^{x-1} + 4$

42. $g(x) = \frac{1}{2} \cdot 5^{1-x} - 2$

43. $g(x) = -e^{x-2} + 3$

44. $g(x) = 3 + e^{2-x}$

45. $g(x) = e^{|x|}$

46. $g(x) = -e^{|x|}$

SECTION 3.2 Exercises

Building Skills

In Exercises 9–20, write each exponential equation in logarithmic form.

9. $5^2 = 25$

10. $(49)^{-1/2} = \frac{1}{7}$

11. $\left(\frac{1}{16}\right)^{-1/2} = 4$

12. $(a^2)^2 = a^4$

13. $10^0 = 1$

14. $10^4 = 10{,}000$

15. $(10)^{-1} = 0.1$

16. $3^x = 5$

17. $a^2 + 2 = 7$

18. $a^e = \pi$

19. $2a^3 - 3 = 10$

20. $5 \cdot 2^{ct} = 11$

In Exercises 21–32, write each logarithmic equation in exponential form.

21. $\log_2 32 = 5$

22. $\log_7 49 = 2$

23. $\log_{10} 100 = 2$

24. $\log_{10} 10 = 1$

25. $\log_{10} 1 = 0$

26. $\log_a 1 = 0$

27. $\log_{10} 0.01 = -2$

28. $\log_{1/5} 5 = -1$

29. $3 \log_8 2 = 1$

30. $1 + \log 1000 = 4$

31. $\ln 2 = x$

32. $\ln \pi = a$

In Exercises 33–42, evaluate each expression without using a calculator.

33. $\log_5 125$

34. $\log_9 81$

35. $\log 10{,}000$

36. $\log_3 \frac{1}{3}$

37. $\log_2 \frac{1}{8}$

38. $\log_4 \frac{1}{64}$

39. $\log_3 \sqrt{27}$

40. $\log_{27} 3$

41. $\log_{16} 2$

42. $\log_5 \sqrt{125}$

In Exercises 43–54, evaluate each expression.

43. $\log_3 1$

44. $\log_{1/2} 1$

45. $\log_7 7$

46. $\log_{1/9} \frac{1}{9}$

47. $\log_6 6^7$

48. $\log_{1/2} \left(\frac{1}{2}\right)^5$

49. $3^{\log_3 5}$

50. $7^{\log_7 \frac{1}{2}}$

51. $2^{\log_2 7} + \log_5 5^{-3}$

52. $3^{\log_3 5} - \log_2 2^{-3}$

53. $4^{\log_4 6} - \log_4 4^{-2}$

54. $10^{\log x} - e^{\ln y}$

In Exercises 55–70, find the domain of each function.

55. $f(x) = \log_2(x + 1)$

56. $g(x) = \log_3(x - 8)$

57. $f(x) = \log_3\sqrt{x - 1}$

58. $g(x) = \log_4\sqrt{3 - x}$

59. $f(x) = \log(x - 2) + \log(2x - 1)$

60. $g(x) = \ln\sqrt{x + 5} - \ln(x + 1)$

61. $h(x) = \ln(x - 1)/\ln(2 - x)$

62. $f(x) = \ln(x - 3) + \ln(2 - x)$

63. $f(x) = \ln|x|$

64. $f(x) = \sqrt{\ln x}$

65. $f(x) = \log_2\left(\frac{x}{3} - 4\right)$

66. $f(x) = \log_3\left(2 - \frac{x}{5}\right)$

67. $f(x) = \ln\left(\frac{x}{x + 1}\right)$

68. $f(x) = \ln\left(\frac{x - 2}{x}\right)$

69. $f(x) = \log_3\left(\frac{x - 2}{x + 1}\right)$

70. $f(x) = \log_2\left(\frac{x + 3}{x - 2}\right)$

71. Match each logarithmic function with one labeled **a–f**.

a. $f(x) = \log x$

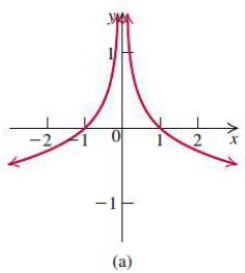
b. $f(x) = -\log|x|$

c. $f(x) = -\log(-x)$

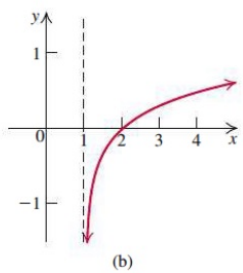
d. $f(x) = \log(x - 1)$

e. $f(x) = (\log x) - 1$

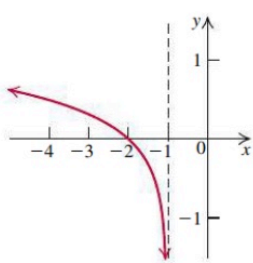
f. $f(x) = \log(-1 - x)$



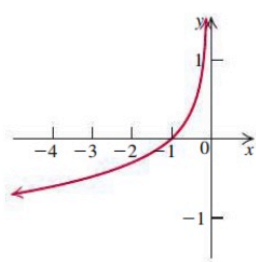
(a)



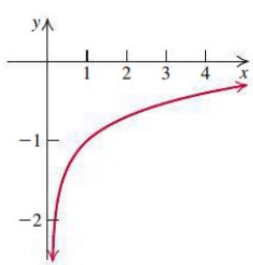
(b)



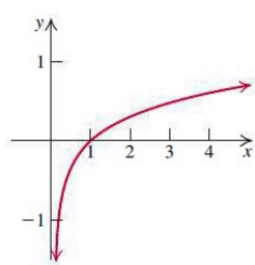
(c)



(d)



(e)



(f)

In Exercises 81–88, begin with the graph of $f(x) = \log_2 x$ and use transformations to sketch the graph of each function. Find the domain and range of the function and the vertical asymptote of the graph.

81. $y = \log_2(x - 1)$

82. $y = \log_2(-x)$

83. $y = \log_2(3 - x)$

84. $y = \log_2 x$

85. $y = 2 + \log_2(3 - x)$

86. $y = 4 - \log_2(3 - x)$

87. $y = \log_2 |x|$

88. $y = \log_2 x^2$

In Exercises 89–94, evaluate each expression.

89. $\log_4(\log_3 81)$

90. $\log_4[\log_3(\log_2 8)]$

91. $\log_{\sqrt{2}} 2$

92. $\log_{2\sqrt{2}} 8$

93. $\log_{\sqrt{2}} 4$

94. $\log_{\sqrt{3}} 27$

In Exercises 95–100, begin with the graph of $y = \ln x$ and use transformations to sketch the graph of each of the given functions.

95. $y = \ln(x + 2)$

96. $y = \ln(2 - x)$

97. $y = -\ln(2 - x)$

98. $y = 2 \ln x$

99. $y = 3 - 2 \ln x$

100. $y = 1 - \ln(1 - x)$

SECTION 3.4 Exercises

Building Skills

In Exercises 9–24, solve each equation.

9. $2^x = 16$

10. $3^x = 243$

11. $8^x = 32$

12. $5^{x-1} = 1$

13. $4^{|x|} = 128$

14. $9^{|x|} = 243$

15. $5^{-|x|} = 625$

16. $3^{-|x|} = 81$

17. $\ln x = 0$

18. $\ln(x - 1) = 1$

19. $\log_2 x = -1$

20. $\log_2(x + 1) = 3$

21. $\log_3 |x| = 2$

22. $\log_2 |x + 1| = 3$

23. $\frac{1}{2}\log x - 2 = 0$

24. $\frac{1}{3}\log(x + 1) - 1 = 0$

In Exercises 25–60, solve each exponential equation. Write the exact answer with natural logarithms and then approximate the result correct to three decimal places.

25. $2^x = 3$

26. $3^x = 5$

27. $2^{2x+3} = 15$

28. $3^{2x+5} = 17$

29. $e^{x+1} = 3$

30. $e^{2x-1} = 5$

31. $5 \cdot 2^x - 7 = 10$

32. $3 \cdot 5^x + 4 = 11$

33. $3 \cdot 4^{2x-1} + 4 = 14$

34. $2 \cdot 3^{4x-5} - 7 = 10$

35. $2e^{x-2} + 3 = 7$

36. $3e^{3x-5} + 1 = 6$

37. $5^{1-x} = 2^x$

38. $3^{2x-1} = 2^{x+1}$

39. $2^{1-x} = 3^{4x+6}$

40. $5^{2x+1} = 3^{x-1}$

41. $2 \cdot 3^{x-1} = 5^{x+1}$

42. $5 \cdot 2^{2x+1} = 7 \cdot 3^{x-1}$

43. $(1.065)^t = 2$

44. $(1.0725)^t = 2$

45. $2^{2x} - 4 \cdot 2^x = 21$

46. $4^x - 4^{-x} = 2$

47. $9^x - 6 \cdot 3^x + 8 = 0$

48. $\dfrac{3^x + 5 \cdot 3^{-x}}{3} = 2$

49. $3^{3x} - 4 \cdot 3^{2x} + 2 \cdot 3^x = 8$

In Exercises 61–78, solve each logarithmic equation.

61. $3 + \log(2x + 5) = 2$

62. $1 + \log(3x - 4) = 0$

63. $\log(x^2 - x - 5) = 0$

64. $\log(x^2 - 6x + 9) = 0$

65. $\log_4(x^2 - 7x + 14) = 1$

66. $\log_4(x^2 + 5x + 10) = 1$

67. $\ln(2x - 3) - \ln(x + 5) = 0$

68. $\log(x + 8) + \log(x - 1) = 1$

69. $\log x + \log(x + 9) = 1$

70. $\log_5(3x - 1) - \log_5(2x + 7) = 0$

71. $\log_a(5x - 2) - \log_a(3x + 4) = 0$

72. $\log(x - 1) + \log(x + 2) = 1$

73. $\log_6(x + 2) + \log_6(x - 3) = 1$

74. $\log_2(3x - 2) - \log_2(5x + 1) = 3$

75. $\log_3(2x - 7) - \log_3(4x - 1) = 2$

76. $\log_4\sqrt{x + 3} - \log_4\sqrt{2x - 1} = \frac{1}{4}$

77. $\log_7 3x + \log_7(2x - 1) = \log_7(16x - 10)$

78. $\log_3(x + 1) + \log_3 2x = \log_3(3x + 1)$