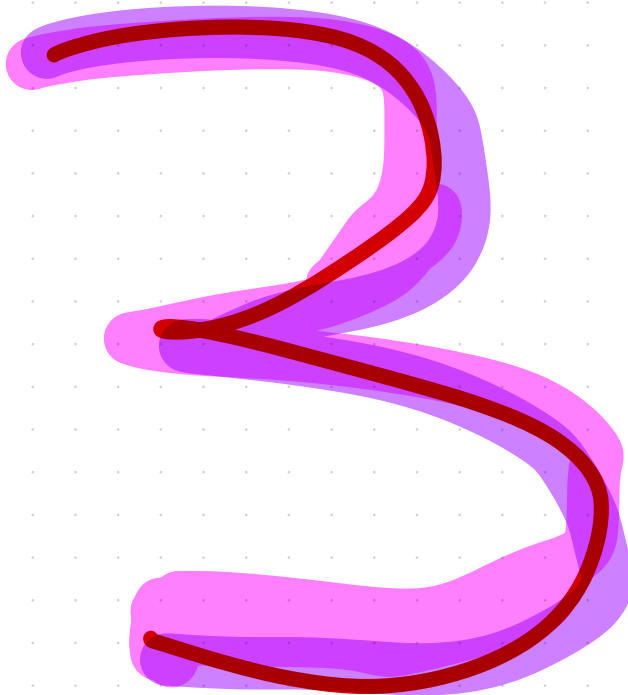

UNIT 3 - Unit circle and trig functions

- 4.1 angle measurements, degrees vs. radians
- 4.2 unit circle, trig functions, the two triangles
- 4.3 reference angles, evaluating trig functions
- 4.4 graphs of sine and cosine, amplitude, period
- 4.5 graphs of tangent, secant, and cosecant
- 4.6 inverse trigonometric functions, evaluating and finding the domain and codomain of inverse trig functions





Building Skills

In Exercises 9–16, draw each angle in standard position.

- | | |
|-----------------------|------------------------|
| 9. 30° | 10. 150° |
| 11. -120° | 12. -330° |
| 13. $\frac{5\pi}{3}$ | 14. $\frac{11\pi}{6}$ |
| 15. $-\frac{4\pi}{3}$ | 16. $-\frac{13\pi}{4}$ |

In Exercises 17–22, convert each angle to decimal degree notation. Round your answers to two decimal places.

- | | |
|--------------------------|--------------------------|
| 17. $70^\circ 45'$ | 18. $38^\circ 38'$ |
| 19. $23^\circ 42' 30''$ | 20. $45^\circ 50' 50''$ |
| 21. $-15^\circ 42' 57''$ | 22. $-70^\circ 18' 13''$ |

In Exercises 23–28, convert each angle to DMS notation. Round your answers to the nearest second.

- | | |
|---------------------|---------------------|
| 23. 27.32° | 24. 120.64° |
| 25. 13.347° | 26. 110.433° |
| 27. 19.0511° | 28. 82.7272° |

In Exercises 29–36, convert each angle from degrees to radians. Express each answer as a multiple of π .

- | | | |
|------------------|------------------|------------------|
| 29. 20° | 30. 40° | 31. -180° |
| 32. -210° | 33. 315° | 34. 330° |
| 35. -510° | 36. -420° | |

In Exercises 37–44, convert each angle from radians to degrees.

- | | | |
|------------------------|----------------------|-----------------------|
| 37. $\frac{\pi}{12}$ | 38. $\frac{3\pi}{8}$ | 39. $-\frac{5\pi}{9}$ |
| 40. $-\frac{3\pi}{10}$ | 41. $\frac{5\pi}{3}$ | 42. $\frac{11\pi}{6}$ |
| 43. $-\frac{11\pi}{4}$ | 44. $\frac{7\pi}{3}$ | |

In Exercises 45–48, convert each angle from degrees to radians. Round your answers to two decimal places.

- | | |
|-----------------|------------------|
| 45. 12° | 46. 127° |
| 47. -84° | 48. -175° |

In Exercises 49–52, convert each angle from radians to degrees. Round your answers to two decimal places.

- | | |
|-------------|-------------|
| 49. 0.94 | 50. 5 |
| 51. -8.21 | 52. -6.28 |

In Exercises 53–58, find the angle between 0 and 2π radians that is coterminal with the given angle.

- | | | |
|-----------------------|-----------------------|-----------------------|
| 53. $-\frac{\pi}{4}$ | 54. $-\frac{2\pi}{3}$ | 55. $-\frac{7\pi}{4}$ |
| 56. $-\frac{5\pi}{3}$ | 57. $\frac{16\pi}{3}$ | 58. $\frac{23\pi}{6}$ |



In Exercises 59–64, find the angle between 0° and 360° that is coterminal with the given angle.

- | | | |
|------------------|------------------|------------------|
| 59. -65° | 60. -120° | 61. -200° |
| 62. -280° | 63. 700° | 64. 1270° |

In Exercises 65–70, find the complement and the supplement of the given angle or explain why the angle has no complement or supplement.

- | | | |
|-----------------|-----------------|-----------------|
| 65. 47° | 66. 75° | 67. 120° |
| 68. 160° | 69. 210° | 70. -50° |

In Exercises 71–90, use the following notations: θ = central angle of a circle, r = radius of a circle, s = length of the intercepted arc, v = linear velocity, ω = angular velocity, A = area of the sector of a circle, and t = time.

In each case, find the missing quantity. Find the exact answer.

71. $r = 25$ in, $s = 7$ in, $\theta = ?$
72. $r = 5$ ft, $s = 6$ ft, $\theta = ?$
73. $r = 10.5$ cm, $s = 22$ cm, $\theta = ?$
74. $r = 60$ m, $s = 120$ m, $\theta = ?$
75. $r = 3$ m, $\theta = 25^\circ$, $s = ?$
76. $r = 0.7$ m, $\theta = 357^\circ$, $s = ?$
77. $r = 6.5$ m, $\theta = 12$ radians, $s = ?$
78. $r = 6$ m, $\theta = \frac{\pi}{6}$ radians, $s = ?$



Building Skills

In Exercises 9–14, determine whether the given point (x, y) is on the unit circle.

9. $\left(\frac{3}{5}, \frac{4}{5}\right)$

10. $\left(\frac{12}{13}, \frac{5}{13}\right)$

11. $\left(\frac{3}{4}, -\frac{\sqrt{7}}{4}\right)$

12. $\left(-\frac{\sqrt{13}}{7}, \frac{6}{7}\right)$

13. $\left(\frac{1}{3}, \frac{2}{3}\right)$

14. $\left(-\frac{3}{4}, \frac{1}{4}\right)$

In Exercises 15–20, find all numbers u (if any) so that the given point (u, y) or (x, u) is on the unit circle.

15. $\left(u, \frac{1}{2}\right)$

16. $\left(u, -\frac{1}{3}\right)$

17. $\left(-\frac{3}{4}, u\right)$

18. $\left(-\frac{2}{5}, u\right)$

19. $\left(u, \frac{3}{2}\right)$

20. $\left(-\frac{4}{3}, u\right)$

In Exercises 21–26, $P(t) = (x, y)$ is the terminal point on the unit circle that corresponds to the real number t . Find the values of $\sin t$, $\cos t$, and $\tan t$.

21. $\left(\frac{2\sqrt{2}}{3}, \frac{1}{3}\right)$

22. $\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

23. $\left(-\frac{1}{3}, \frac{2\sqrt{2}}{3}\right)$

24. $\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

25. $\left(\frac{1}{5}, -\frac{2\sqrt{6}}{5}\right)$

26. $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$

In Exercises 27–32, let $P(t) = (x, y)$. Find the values of $\sec t$, $\csc t$, and $\cot t$.

27. $\left(\frac{3}{4}, \frac{\sqrt{7}}{4}\right)$

28. $\left(\frac{5}{6}, -\frac{\sqrt{11}}{6}\right)$

29. $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{6}}{3}\right)$

30. $\left(-\frac{2\sqrt{2}}{3}, -\frac{1}{3}\right)$

31. $\left(-\frac{2\sqrt{6}}{5}, -\frac{1}{5}\right)$

32. $\left(-\frac{2}{7}, \frac{3\sqrt{5}}{7}\right)$

In Exercises 33–38, find the values (if any) of the six trigonometric functions of each value of t .

33. $t = 5\pi$

34. $t = 3\pi$

35. $t = -\frac{3\pi}{2}$

36. $t = \frac{5\pi}{2}$

37. $t = \frac{7\pi}{2}$

38. $t = -\frac{9\pi}{2}$

In Exercises 39–50, find each trigonometric function value.

39. $\tan(4\pi)$

40. $\sec(7\pi)$

41. $\sin(-2\pi)$

42. $\cos(-5\pi)$

43. $\cos\left(-\frac{3\pi}{2}\right)$

44. $\sin\left(-\frac{\pi}{2}\right)$

45. $\sin(-180^\circ)$ 46. $\cos(360^\circ)$ 47. $\tan(540^\circ)$
48. $\sec(270^\circ)$ 49. $\csc(-720^\circ)$ 50. $\cot(-630^\circ)$

In Exercises 51–62, use Table 4.1 to find the exact value of each expression.

51. $\sin 180^\circ - \cos 90^\circ$ 52. $\cos 180^\circ - \sin 90^\circ$
53. $\sin 30^\circ \sec 60^\circ$ 54. $\sin 270^\circ \csc 45^\circ$
55. $2 \tan 60^\circ \cos 30^\circ - \cot 30^\circ \csc 60^\circ$
56. $3 \tan 30^\circ \cot 60^\circ + \csc 30^\circ \cos 60^\circ$
57. $\sin \frac{\pi}{4} - \cos \pi$ 58. $\sec \pi + \sin \frac{\pi}{6}$
59. $\tan \frac{\pi}{4} - \cot \frac{\pi}{3}$ 60. $\csc \frac{\pi}{2} + \cos \frac{\pi}{3}$
61. $\sin \frac{3\pi}{2} \tan \frac{\pi}{4}$ 62. $\cos \frac{\pi}{2} \sec \pi$

In Exercises 63–92, use Table 4.1 and symmetry (see Figure 4.32) to find the exact value of each expression.

63. $\sin\left(\frac{7\pi}{6}\right)$ 64. $\cos\left(\frac{11\pi}{6}\right)$ 65. $\tan\left(\frac{2\pi}{3}\right)$
66. $\cot\left(\frac{3\pi}{4}\right)$ 67. $\sec\left(\frac{4\pi}{3}\right)$ 68. $\tan\left(\frac{7\pi}{6}\right)$
69. $\csc\left(\frac{11\pi}{6}\right)$ 70. $\sin\left(\frac{7\pi}{4}\right)$ 71. $\cos\left(\frac{5\pi}{3}\right)$
72. $\sin 120^\circ$ 73. $\cos 135^\circ$ 74. $\sec 150^\circ$
75. $\tan 210^\circ$ 76. $\sin 225^\circ$ 77. $\cot 240^\circ$
78. $\sin 300^\circ$ 79. $\cos 315^\circ$ 80. $\tan 330^\circ$
81. $\sin\left(-\frac{5\pi}{6}\right)$ 82. $\cos\left(-\frac{5\pi}{4}\right)$ 83. $\tan\left(-\frac{2\pi}{3}\right)$
84. $\sec\left(-\frac{7\pi}{6}\right)$ 85. $\csc\left(-\frac{5\pi}{3}\right)$ 86. $\tan\left(-\frac{7\pi}{4}\right)$
87. $\sin(-30^\circ)$ 88. $\cos(-45^\circ)$ 89. $\tan(-120^\circ)$
90. $\sec(-150^\circ)$ 91. $\csc(-240^\circ)$ 92. $\cot(-300^\circ)$

**Building Skills**

In Exercises 9–14, a point on the terminal side of an angle θ in standard position is given. Find the exact values of $\sin \theta$, $\cos \theta$, and $\tan \theta$.

9. $(-4, 3)$ 10. $(-3, 5)$
11. $(-\sqrt{3}, -1)$ 12. $(-1, -2)$
13. $(3, 3)$ 14. $(-2, -2)$

In Exercises 15–20, a point on the terminal side of an angle θ in standard position is given. Find the exact value of $\csc \theta$, $\sec \theta$, and $\cot \theta$.

15. $(12, -5)$ 16. $(7, -2)$
17. $(7, -24)$ 18. $(5, -5)$
19. $(-\sqrt{2}, \sqrt{6})$ 20. $(-\sqrt{5}, -\sqrt{11})$

In Exercises 21–28, use the given information to find the quadrant in which each angle θ lies.

21. $\sin \theta < 0$ and $\cos \theta < 0$ 22. $\sin \theta < 0$ and $\tan \theta > 0$
23. $\sin \theta > 0$ and $\cos \theta < 0$ 24. $\tan \theta > 0$ and $\csc \theta < 0$
25. $\cos \theta > 0$ and $\csc \theta < 0$ 26. $\cos \theta < 0$ and $\cot \theta > 0$
27. $\sec \theta < 0$ and $\csc \theta > 0$ 28. $\sec \theta < 0$ and $\tan \theta > 0$

In Exercises 29–36, find the exact values of the trigonometric functions of θ from the given information.

29. $\cos \theta = -\frac{5}{13}$, θ in quadrant III, find $\tan \theta$.



30. $\tan \theta = -\frac{3}{4}$, θ in quadrant IV, find $\sin \theta$.

31. $\cot \theta = -\frac{3}{4}$, θ in quadrant II, find $\cos \theta$.

32. $\sec \theta = \frac{4}{\sqrt{7}}$, θ in quadrant IV, find $\csc \theta$.

33. $\sin \theta = \frac{3}{5}$, $\tan \theta < 0$, find $\sec \theta$.

34. $\cot \theta = \frac{3}{2}$, $\sec \theta > 0$, find $\sin \theta$.

35. $\sec \theta = 3$, $\sin \theta < 0$, find $\cot \theta$.

36. $\tan \theta = -2$, $\sin \theta > 0$, find $\cos \theta$.

In Exercises 37–48, find the reference angle for each angle.

37. 120° 38. 275° 39. -50°

40. 500° 41. 420° 42. -110°

43. $\frac{19\pi}{4}$ 44. $\frac{28\pi}{6}$ 45. $-\frac{3\pi}{4}$

46. $\frac{31\pi}{6}$ 47. $\frac{5\pi}{6}$ 48. $-\frac{15\pi}{4}$



In Exercises 49–60, use the reference angle to find the exact value of each expression.

49. $\cos 120^\circ$

50. $\sin 315^\circ$

51. $\tan 510^\circ$

52. $\cot 750^\circ$

53. $\sec 210^\circ$

54. $\csc 300^\circ$

55. $\sin \frac{7\pi}{6}$

56. $\cos \frac{4\pi}{3}$

57. $\sec \frac{15\pi}{4}$

58. $\cot \frac{19\pi}{4}$

59. $\sin \frac{29\pi}{3}$

60. $\tan \frac{55\pi}{3}$

In Exercises 61–66, use reciprocal and quotient identities to find the exact value of each expression.

61. Given $\sin t = \frac{1}{3}$ and $\cos t = \frac{2\sqrt{2}}{3}$, find $\tan t$.

62. Given $\sin t = -\frac{1}{4}$ and $\cos t = -\frac{\sqrt{15}}{2}$, find $\cot t$.

63. Given $\sin t = \frac{2}{3}$ and $\cot t = -\frac{\sqrt{5}}{2}$, find $\cos t$.

64. Given $\cos t = -\frac{2}{5}$ and $\tan t = -\frac{\sqrt{21}}{2}$, find $\sin t$.

65. Given $\csc t = \frac{7}{2}$ and $\cos t = -\frac{3\sqrt{5}}{7}$, find $\tan t$.

66. Given $\csc t = \frac{8}{3}$ and $\sec t = \frac{8\sqrt{55}}{55}$, find $\cot t$.

In Exercises 67–78, use fundamental identities to find the exact value of each expression.

67. $\sin 70^\circ \csc 70^\circ$

68. $\tan 65^\circ \cot 65^\circ$

69. $\cos 35^\circ \sec 35^\circ$

70. $\sin^2 \frac{2\pi}{9} + \cos^2 \frac{2\pi}{9}$

71. $\cos^2 47^\circ + \sin^2 47^\circ$

72. $\sec^2 \frac{5\pi}{12} - \tan^2 \frac{5\pi}{12}$

73. $\sin^2 \frac{\pi}{7} + \cos^2 \frac{\pi}{7} + \cot^2 \frac{\pi}{7}$

74. $\csc^2 32^\circ - \cot^2 32^\circ$

75. $\tan \frac{3\pi}{13} - \frac{\sin \frac{3\pi}{13}}{\cos \frac{3\pi}{13}}$

76. $\cot 76^\circ - \frac{\cos 76^\circ}{\sin 76^\circ}$

77. $\cos 390^\circ \sec 30^\circ$

78. $\sin \frac{3\pi}{11} \csc \frac{47\pi}{11}$

In Exercises 79–86, use the Pythagorean identities to find the exact value of each expression.

79. Given $\sin t = \frac{2}{5}$ and $\cos t < 0$, find $\cos t$.

80. Given $\cos t = \frac{1}{6}$ and $\sin t < 0$, find $\sin t$.

81. Given $\sec t = -5$ and $\tan t < 0$, find $\tan t$.

82. Given $\cot t = 4$ and $\csc t < 0$, find $\csc t$.

83. Given $\tan t = -3$ and $\sec t > 0$, find $\sec t$.



In Exercises 9–28, sketch the graph of each given equation over the interval $[-2\pi, 2\pi]$.

9. $y = 2 \sin x$

10. $y = 4 \cos x$

11. $y = -\frac{1}{2} \sin x$

12. $y = -2 \sin x$

13. $y = \frac{3}{2} \cos x$

14. $y = \frac{5}{4} \sin x$

15. $y = \cos 2x$

16. $y = \sin 4x$

17. $y = \cos \frac{2}{3}x$

18. $y = \sin \frac{4}{3}x$

19. $y = \cos\left(x + \frac{\pi}{2}\right)$

20. $y = \sin\left(x + \frac{\pi}{4}\right)$

21. $y = \cos\left(x - \frac{\pi}{3}\right)$

22. $y = \sin(x - \pi)$

23. $y = 2 \cos\left(x - \frac{\pi}{2}\right)$

24. $y = 2 \sin\left(x + \frac{\pi}{3}\right)$

25. $y = \sin x + 1$

26. $y = \cos x - 2$

27. $y = -\cos x + 1$

28. $y = \sin x - 3$

In Exercises 29–36, find the amplitude, period, and phase shift of each given function.

29. $y = 5 \cos(x - \pi)$

30. $y = 3 \sin\left(x - \frac{\pi}{8}\right)$

31. $y = 7 \cos\left[9\left(x + \frac{\pi}{6}\right)\right]$

32. $y = 11 \sin\left[8\left(x + \frac{\pi}{3}\right)\right]$

33. $y = -6 \cos\left[\frac{1}{2}(x + 2)\right]$

34. $y = -8 \sin\left[\frac{1}{5}(x + 9)\right]$

35. $y = 0.9 \sin\left[0.25\left(x - \frac{\pi}{4}\right)\right]$

36. $y = \sqrt{5} \cos[\pi(x + 1)]$

**Building Skills**

In Exercises 9–12, find the slope–intercept form of the equation of each line that passes through the point P and makes angle θ with the positive x -axis.

9. $P = (-2, 3), \theta = 45^\circ$



10. $P = (3, -1), \theta = 60^\circ$

11. $P = (-3, -2), \theta = 120^\circ$

12. $P = (2, 5), \theta = 135^\circ$

In Exercises 13–30, graph each function over a one-period interval.

13. $y = \tan\left(x - \frac{\pi}{4}\right)$

14. $y = \tan\left(x + \frac{\pi}{4}\right)$

15. $y = \cot\left(x + \frac{\pi}{4}\right)$

16. $y = \cot\left(x - \frac{\pi}{4}\right)$

17. $y = \tan 2x$

18. $y = \tan \frac{x}{2}$

19. $y = \cot \frac{x}{2}$

20. $y = \cot 2x$

21. $y = -\tan x$

22. $y = -\cot x$

23. $y = 3 \tan x$

24. $y = 3 \cot x$

25. $y = \sec \frac{x}{2}$

26. $y = \sec 2x$

27. $y = \csc 3x$

28. $y = \csc \frac{x}{3}$

29. $y = \sec(x - \pi)$

30. $y = \csc(x - \pi)$

In Exercises 31–48, graph each function over a two-period interval.

31. $y = \tan\left[2\left(x + \frac{\pi}{2}\right)\right]$

32. $y = \tan\left[2\left(x - \frac{\pi}{2}\right)\right]$

33. $y = \cot\left[2\left(x - \frac{\pi}{2}\right)\right]$

34. $y = \cot\left[2\left(x + \frac{\pi}{2}\right)\right]$

35. $y = \tan\left[\frac{1}{2}(x + 2\pi)\right]$

36. $y = \tan\left[\frac{1}{2}(x - 2\pi)\right]$

37. $y = \cot\left[\frac{1}{2}(x - 2\pi)\right]$

38. $y = \cot\left[\frac{1}{2}(x + 2\pi)\right]$

39. $y = \sec\left[4\left(x - \frac{\pi}{4}\right)\right]$

40. $y = \sec\left[\frac{1}{2}\left(x - \frac{\pi}{2}\right)\right]$

41. $y = 3 \csc\left(x + \frac{\pi}{2}\right)$

42. $y = 3 \sec\left[2\left(x - \frac{\pi}{6}\right)\right]$

43. $y = \tan\left[\frac{2}{3}\left(x - \frac{\pi}{2}\right)\right]$

44. $y = 2 \cot\left[2\left(x - \frac{\pi}{6}\right)\right]$

45. $y = -5 \tan\left[2\left(x + \frac{\pi}{3}\right)\right]$

46. $y = -3 \cot\left[\frac{1}{2}\left(x - \frac{\pi}{3}\right)\right]$



Building Skills

In Exercises 9–30, find each exact value of y or state that y is undefined.

- | | |
|--|---|
| 9. $y = \sin^{-1} 0$ | 10. $y = \cos^{-1} 0$ |
| 11. $y = \sin^{-1}\left(-\frac{1}{2}\right)$ | 12. $y = \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$ |
| 13. $y = \arccos(-1)$ | 14. $y = \arcsin \frac{1}{2}$ |
| 15. $y = \arccos \frac{\pi}{2}$ | 16. $y = \arcsin \pi$ |
| 17. $y = \tan^{-1} \sqrt{3}$ | 18. $y = \tan^{-1} 1$ |
| 19. $y = \arctan(-1)$ | 20. $y = \arctan\left(-\frac{\sqrt{3}}{3}\right)$ |
| 21. $y = \cot^{-1}(-1)$ | 22. $y = \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$ |
| 23. $y = \cos^{-1}(-2)$ | 24. $y = \sin^{-1} \sqrt{3}$ |
| 25. $y = \sec^{-1}(-2)$ | 26. $y = \csc^{-1}(-2)$ |
| 27. $y = \arcsin 1$ | 28. $y = \arccos 1$ |
| 29. $y = \operatorname{arccot}(-\sqrt{3})$ | 30. $y = \operatorname{arcsec}(-\sqrt{2})$ |

In Exercises 31–36, use the four-step procedure (page 122) to find $f^{-1}(x)$ for the given one-to-one function $f(x)$.

31. $f(x) = 2 \sin x + 1, -\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$
32. $f(x) = \frac{1}{2} \cos x - 1, 0 \leq x \leq \pi$
33. $f(x) = 3 \cos(2x - 1), \frac{1}{2} \leq x \leq \frac{\pi}{2} + \frac{1}{2}$
34. $f(x) = 4 \sin(3x - 2), -\frac{\pi}{6} + \frac{2}{3} \leq x \leq \frac{\pi}{6} + \frac{2}{3}$
35. $f(x) = \tan(x - 1) + 2, 1 - \frac{\pi}{2} < x < 1 + \frac{\pi}{2}$
36. $f(x) = \cot(x + 1) - 3, -1 \leq x \leq \pi - 1$

In Exercises 37–52, find each exact value of y or state that y is undefined.

- | | |
|---|---|
| 37. $y = \sin\left(\sin^{-1} \frac{1}{8}\right)$ | 38. $y = \cos\left(\cos^{-1} \frac{1}{5}\right)$ |
| 39. $y = \tan^{-1}\left(\tan \frac{\pi}{7}\right)$ | 40. $y = \tan^{-1}\left(\tan \frac{\pi}{4}\right)$ |
| 41. $y = \tan(\tan^{-1} 247)$ | 42. $y = \tan(\tan^{-1} 7)$ |
| 43. $y = \sin^{-1}\left(\sin \frac{4\pi}{3}\right)$ | 44. $y = \cos^{-1}\left(\cos \frac{5\pi}{3}\right)$ |
| 45. $y = \tan^{-1}\left(\tan \frac{2\pi}{3}\right)$ | 46. $y = \tan\left(\tan^{-1} \frac{2\pi}{3}\right)$ |
| 47. $y = \sin^{-1}\left(\sin \frac{3\pi}{4}\right)$ | 48. $y = \cos^{-1}\left(\cos \frac{7\pi}{6}\right)$ |
| 49. $y = \sin(\sin^{-1} \sqrt{2})$ | 50. $y = \cos(\cos^{-1}(-\sqrt{2}))$ |
| 51. $y = \cos^{-1}(\cos(-\pi))$ | 52. $y = \sin^{-1}(\sin 1.2)$ |

In Exercises 53–68, use the identities on page 422 to find the exact value of each expression.

- | | |
|---|---|
| 53. $\cot^{-1}\left(\frac{1}{\sqrt{3}}\right)$ | 54. $\sec^{-1}(\sqrt{2})$ |
| 55. $\csc^{-1}(2)$ | 56. $\csc^{-1}\left(\frac{2\sqrt{3}}{3}\right)$ |
| 57. $\cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)$ | 58. $\cot^{-1}(-\sqrt{3})$ |



59. $\sin^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

60. $\csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

61. $\cos^{-1}\left(-\frac{1}{2}\right)$

62. $\sec^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

63. $\sin\left[\frac{\pi}{3} - \sin^{-1}\left(-\frac{1}{2}\right)\right]$

64. $\cos\left[\frac{\pi}{6} + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

65. $\sin\left[\frac{\pi}{2} - \cos^{-1}(-1)\right]$

66. $\tan\left[\frac{\pi}{6} + \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right)\right]$

67. $\sin\left[\tan^{-1}(-\sqrt{3}) + \cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)\right]$

68. $\cos\left[\cot^{-1}(-\sqrt{3}) + \sin^{-1}\left(-\frac{1}{2}\right)\right]$

In Exercises 79–92, use a sketch to find each exact value of y .

79. $y = \cos\left(\sin^{-1}\frac{2}{3}\right)$

80. $y = \sin\left(\cos^{-1}\frac{3}{4}\right)$

81. $y = \sin\left[\cos^{-1}\left(-\frac{4}{5}\right)\right]$

82. $y = \cos\left(\sin^{-1}\frac{3}{5}\right)$

83. $y = \cos\left(\tan^{-1}\frac{5}{2}\right)$

84. $y = \sin\left(\tan^{-1}\frac{13}{5}\right)$

85. $y = \tan\left(\cos^{-1}\frac{4}{5}\right)$

86. $y = \tan\left[\sin^{-1}\left(-\frac{3}{4}\right)\right]$

87. $y = \sin(\tan^{-1} 4)$

88. $y = \cos(\tan^{-1} 3)$

89. $y = \tan(\sec^{-1} 2)$

90. $y = \tan[\csc^{-1}(-2)]$

91. $y = \sin(\cos^{-1}x), |x| < 1$

92. $\tan\left(\sin^{-1}\frac{x}{4}\right), |x| < 4$