Exam 1 Key

1. Find the inverse of the function $f(x) = \frac{x+3}{x-2}$, and give the domain and range of f and f^{-1}.

(12 pts.)

$$x = \frac{y+3}{y-2}$$

$$f^{-1}(x) = \frac{2x+3}{x-1}$$

$$x(y-2) = y+3$$

$$xy - 2x = y+3$$

$$xy - y = 2x+3$$

$$y(x-1) = 2x+3$$

$$y = \frac{2x+3}{x-1}$$

Dom f : $x \neq 2$ Dom f^{-1} : $x \neq 1$

Range f : $y \neq 1$ Range f^{-1} : $y \neq 2$

2. Let $f(x) = \sqrt{x-4}$ and $g(x) = 2-3x$. Find $(f \circ g)(x)$ and also find the domain of $(f \circ g)(x)$.

(12 pts.)

$$f \circ g(x) = f(g(x)) = f(2-3x) = \sqrt{2-3x-4}$$

$$f \circ g(x) = \sqrt{-2-3x}$$

Domain of $f \circ g$

$$-2 - 3x \geq 0 \quad \Big\} +$$

So $-3x \geq 2$

$$\boxed{x \leq -\frac{2}{3}}$$

1

3. Write an equation for a function whose graph fits the description given as follows. The graph of $f(x) = |x|$ is shifted 2 units right, and shifted 3 units down. (8 pts.)

$$y = |x - 2| - 3$$

4. Let $f(x) = 3x^2 - 2x$. Find $f(-3)$ and $f(2)$ and find the average rate of change as x changes from -3 to 2. (12 pts.)

$$f(-3) = 3(-3)^2 - 2(-3) = 27 + 6 = 33$$
$$f(2) = 3(2)^2 - 2(2) = 12 - 4 = 8$$

$$\text{Avg RoC over } [-3,2] = \frac{f(3) - f(2)}{-3 - 2} = \frac{33 - 8}{-5} = \frac{-25}{-5} = \boxed{5}$$

5. Find an equation of the vertical line through the point $(2, 6)$. (8 pts.)

$$\boxed{x = 2}$$

2

6. It costs $15 flat fee to rent a lawnmower, plus $3 an hour starting with the first hour. Let x represent the number of hours rented, so y represents the charge to the user. **Write an equation** of the form $y = mx + b$ that represents the situation. If the user was charged $24, then **how many hours** did they rent the mower? (8 pts.)

$$y = 15 + 3x$$

$$24 = 15 + 3x$$

$$9 = 3x$$

$$x = 3 \quad \text{So} \quad \boxed{3 \text{ hours}}$$

7. Write an equation of the line containing the given point and parallel to the given line. (10 pts.)

$$(2, -3); \quad 2x - 3y = 4$$

$$2x - 3y = 4$$

$$\Leftrightarrow \quad -3y = 4 - 2x$$

$$\Leftrightarrow \quad y = -\frac{4}{3} + \frac{2}{3}x$$

$$\text{slope} \quad \frac{2}{3}$$

$$y - y_1 = m(x - x_1)$$

$$y - (-3) = \frac{2}{3}(x - 2)$$

$$\boxed{y + 3 = \frac{2}{3}(x - 2)}$$

or

$$y = \frac{2}{3}x - \frac{4}{3} - 3 \quad \text{or} \quad \boxed{y = \frac{2}{3}x - \frac{13}{3}}$$

8. Simplify the expression $\dfrac{\frac{2}{x+h} - \frac{2}{x}}{h}$. (12 pts.)

$$\left(\frac{2}{x+h} - \frac{2}{x} \right) \frac{1}{h}$$

$$= \left(\frac{2x - 2(x + h)}{x(x+h)} \right) \frac{1}{h}$$

$$= \frac{2x - 2x - 2h}{x(x+h)} \cdot \frac{1}{h}$$

$$= \frac{-2h}{x(x+h)} \cdot \frac{1}{h}$$

$$= \boxed{\frac{-2}{x(x+h)}}$$

3

9. What is the center and radius of a circle with endpoints of a diameter at the points $(3, 5)$ and $(-1, 1)$? (12 pts.)

diameter $d = \sqrt{(3-(-1))^2 + (5-1)^2}$

$= \sqrt{4^2 + 4^2} = \sqrt{32} = 4\sqrt{2}$

radius $= \frac{1}{2}$ diameter $= 4\sqrt{2} * \frac{1}{2} = \boxed{2\sqrt{2}}$ radius

Center midpoint $x = \frac{3+(-1)}{2} = 1$ center at

$y = \frac{5+1}{2} = 3$ $\boxed{(1, 3)}$

10. Simplify the expression $(-8)^{1/3}$. (6 pts.)

$(-8)^{1/3} = \boxed{-2}$

4