

Name: Key

GTID: _____

For True/False problems: Fill in the circle completely for the correct answer. If the sentence is False, put the correct expression for the underlined portion in the box.

For everything else: Show all work and BOX your final answer for each problem. Answers with no work may receive partial or no credit.

1. (1 point) True or False: The domain of the following function: $f(x) = \frac{1}{\sqrt{4-x^2}}$ is $[0, 2]$.

 True False

(-2, 2)

2. (1 point) True or False: The range of f^{-1} equals the domain of f .

 True False

3. (1 point) True or False: If no vertical line intersects the graph of a relation at more than one point, then the graph of the relation is the graph of a one-to-one function.

 True False

function

4. (1 point) Fill in the blank: The graph of an even function is symmetric with respect to

the y-axis.

5. (1 point) Fill in the blank: The graph of $y = -f(x)$ is found by reflecting the graph

$y = f(x)$ about the x-axis.

6. (4 points) Simplify: $\left(\frac{6}{y^2}\right) \left(\frac{-3x^2y^{-2}}{x^3}\right)^{-3}$

$$= \left(\frac{6}{y^2}\right) \frac{(-3)^{-3} x^{-6} y^6}{x^{-9}}$$

$$= \frac{6}{y^2} \cdot \frac{x^9 y^6}{-27 x^6}$$

$$= \frac{6x^3 y^4}{-27} = \boxed{\frac{-2}{9} x^3 y^4}$$

7. (4 points) Simplify: $\frac{-4}{3 - (1 - 2i)}$

$$= \frac{-4}{3 - 1 + 2i} = \frac{-4}{2 + 2i} \cdot \frac{2 - 2i}{2 - 2i} = \frac{-8 + 8i}{4 - 4i^2}$$

$$= \frac{-8 + 8i}{4 + 4}$$

$$= \frac{-8 + 8i}{8}$$

$$= \boxed{-1 + i}$$

8. (5 points) Find the equation in slope-intercept form of the line which is parallel to the line $x + 2y = 4$ and contains the point $(-4, 3)$.

$$x + 2y = 4$$

$$y = -\frac{1}{2}x + b \quad @ \quad (-4, 3)$$

$$\Rightarrow 2y = -x + 4$$

$$\Rightarrow 3 = -\frac{1}{2}(-4) + b$$

$$\Rightarrow y = -\frac{1}{2}x + 2$$

$$\Rightarrow 3 = 2 + b$$

$$m = -1/2$$

$$\Rightarrow b = 1$$

$$y = -\frac{1}{2}x + 1$$

9. (6 points) Given $g(x) = \sqrt{x} + 1$ and $h(x) = \frac{1}{1-2x}$, find (a) the output values of the composite function $f(x) = (g \circ h)(x) = g(h(x))$ at input values $a = -4$ and $b = 0$, and (b) the average rate of change of $f(x)$ as x changes from $a = -4$ to $b = 0$.

Hint: for part (b) use the formula $\frac{f(b)-f(a)}{b-a}$.

(a) $f(x) = g(h(x))$ at $a = -4$

$$h(-4) = \frac{1}{1-2(-4)} = \frac{1}{1+8} = \frac{1}{9}$$

$$g\left(\frac{1}{9}\right) = \sqrt{\frac{1}{9}} + 1 = \frac{1}{3} + 1 = \underline{\underline{\frac{4}{3}}}$$

$f(x)$ at $b = 0$

$$h(-1) = \frac{1}{1-2(0)} = \frac{1}{1+0} = 1$$

$$g(1) = \sqrt{1} + 1 = \underline{\underline{2}}$$

(b) $\frac{f(b)-f(a)}{b-a}$

$$= \frac{2 - \frac{4}{3}}{4 - 0}$$

$$= \frac{\frac{2}{3}}{4}$$

$$= \frac{2}{3} \cdot \frac{1}{4} = \underline{\underline{\frac{1}{6}}}$$

10. (5 points) Find the x and y intercepts of $y = \frac{4x + 8}{2 - 5x}$.

y -intercept where $x=0$.

$$\text{So } \frac{4(0) + 8}{2 - 5(0)} = \frac{8}{2} = 4$$

y -intercept @ $(0, 4)$
 x -intercept @ $(-2, 0)$

x -intercept where $y=0$

So set

$$\frac{4x + 8}{2 - 5x} = 0 \Rightarrow 4x + 8 = 0 \Rightarrow x = -2$$

11. (6 points) Find the inverse of f , state the domain and range of f , and state the domain and range of f^{-1} :

$$f(x) = \frac{x}{2x - 1}$$

Swap x & y & solve

$$x = \frac{y}{2y - 1}$$

$$\Rightarrow x(2y - 1) = y$$

$$\Rightarrow 2xy - x = y$$

$$\Rightarrow 2xy - y = x$$

$$\Rightarrow y(2x - 1) = x \quad \uparrow$$

$$\Rightarrow y = \frac{x}{2x - 1} = f(x) = f^{-1}(x)$$

$D: x \neq \frac{1}{2}$
 $R: x \neq \frac{1}{2}$

12. (6 points) Solve the system and classify the type of solutions (*show all steps for full credit*):

$$\begin{cases} 2x - 3y = 5 \\ 3x - 4y = 8 \end{cases}$$

$$\Rightarrow \begin{array}{l} 3R_1 \rightarrow \\ -2R_2 \rightarrow \end{array} \begin{cases} 6x - 9y = 15 \\ -6x + 8y = -16 \end{cases}$$

$$0x - y = -1$$

$$2x - 3(1) = 5$$

$$-y = -1 \Rightarrow \underline{\underline{y = 1}}$$

$$2x = 8$$

$$\underline{\underline{x = 4}}$$

unique
Solution
@ (4, 1)

13. (3 points) Find $f(-2)$:

$$f(x) = \begin{cases} 3x - 2, & x < -5 \\ \frac{1-x^2}{x}, & -5 \leq x < -1 \\ |x| + 1, & x \geq -1 \end{cases}$$

-2 is in the range $-5 \leq x < -1$

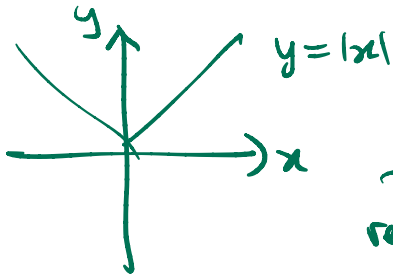
so plug in $x = -2$ into $\frac{1-x^2}{x}$

and get

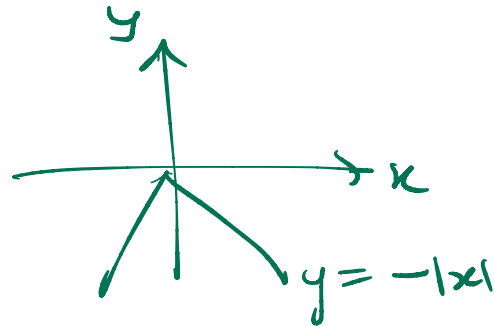
$$f(-2) = \frac{1 - (-2)^2}{-2} = \frac{1 - 4}{-2} = \frac{-3}{-2} = \frac{3}{2}$$

14. (6 points) Sketch the function $f(x)$ and determine whether $y = f(x)$ is EVEN, ODD, or neither:

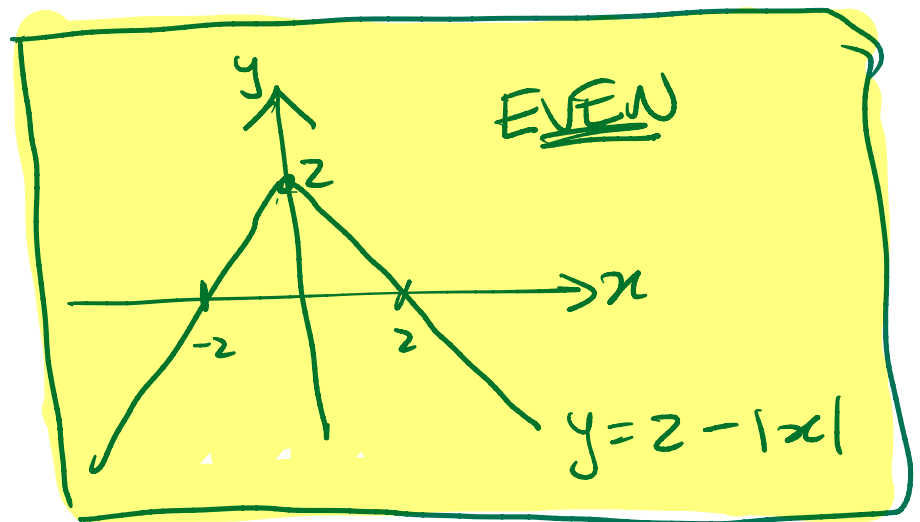
$$f(x) = 2 - |x|$$



reflect
about
x-axis



Shift
up
2



Function is EVEN

Since $f(x) = 2 - |x| \checkmark$

and $f(-x) = 2 - |-x|$
 $= 2 - |x| \checkmark$