

Name: Key

GTID: \_\_\_\_\_

For the short answer problems on **this page** you do not need to show any work.

For everything else: Show all work and BOX your final answer for each problem. Answers with no work may receive partial or no credit. Simplify answers for full credit.

- 1a. (2 points) What is the domain and range of  $y = 3(x - 4)^2 - 5$ ?

D:

$$(-\infty, \infty)$$

R:

$$[-5, \infty)$$

- 1b. (2 points) What is the domain and range of  $y = 2^{x-3} + 4$ ?

D:

$$(-\infty, \infty)$$

R:

$$(4, \infty)$$

- 1c. (2 points) Write the equation for the function which is obtained from the graph of  $y = 4^x$  after being transformed by shifting 2 units up and shifting 1 units left.

$$y = 4^{x+1} + 2$$

2. (5 points) Write in standard form (*i.e.*, vertex form):  $y = 2x^2 - 16x + 22$

$$y = 2(x^2 - 8x) + 22$$

$$\begin{aligned} \Rightarrow y &= 2(x^2 - 8x + 16 - 16) + 22 \\ &= 2(x^2 - 8x + 16) - 32 + 22 \end{aligned}$$

$$y = 2(x - 4)^2 - 10$$

3. (5 points) Find the quotient:  $(x^3 - x^2 - 10x + 6) \div (x + 3)$   
*Hint: check your answer.*

$$\begin{array}{r} x^2 - 4x + 2 \quad R.0 \\ x+3 \overline{) x^3 - x^2 - 10x + 6} \\ \underline{-(x^3 + 3x^2)} \phantom{+ 6} \\ -4x^2 - 10x + 6 \\ \underline{-(-4x^2 - 12x)} \phantom{+ 6} \\ 2x + 6 \\ \underline{-(2x + 6)} \\ 0 \end{array}$$

quotient is

$$x^2 - 4x + 2$$

4. (6 points) For this problem use the function  $f(x) = \frac{x^2 - 4}{(3x - 1)(x + 1)}$ .

Find the domain of  $y = f(x)$ , the VA(s), the HA, the  $x$ -intercept(s), and the  $y$ -intercept.

domain is where denominator is nonzero

$$(3x - 1)(x + 1) = 0$$

$$\Rightarrow 3x - 1 = 0 \Rightarrow x = 1/3$$

or  $x + 1 = 0$       or  $x = -1$

VA is where denominator is zero  
but numerator is nonzero (no hole)

HA is leading coeff of numerator  
 $\div$  leading coeff of denominator

(if deg is same)

$$\text{So } \frac{1x^2 - 4}{(3x - 1)(x + 1)} = \frac{1x^2 - 4}{3x^2 + 2x - 1}$$

$$\text{so } \boxed{y = 1/3}$$

$x$ -intercept is where numerator is zero

$$x^2 - 4 = 0 \Rightarrow (x - 2)(x + 2) = 0 \Rightarrow x = \pm 2$$

$y$ -intercept is where  $x = 0$

$$\frac{0 - 4}{(0 - 1)(0 + 1)} = \frac{-4}{-1} = 4$$

$$\text{so } y = 4$$

D:

$$(-\infty, -1) \cup (-1, 1/3) \cup (1/3, \infty)$$

VA(s):

$$x = 1/3 \text{ and } x = -1$$

HA:

$$y = 1/3$$

$x$ -int:

$$x = 2 \text{ and } x = -2$$

$y$ -int:

$$y = 4$$

5. (4 points) Evaluate:  $\log_2(\sqrt{8})$

$$\begin{aligned}\log_2(\sqrt{8}) &= \log_2(8^{1/2}) \\ &= \frac{1}{2} \log_2(8) = \frac{1}{2} * 3 = \boxed{3/2}\end{aligned}$$

and  $\log_2(8) = y \iff 2^y = 8$  so  $y = 3$ .

6. (4 points) Solve:  $\log_3(2x + 4) = 0$

$$\log_3(y) = 0 \Rightarrow y = 1.$$

$$\text{so } 2x + 4 = 1$$

$$\Rightarrow 2x = -3$$

$$\Rightarrow \boxed{x = -3/2}$$

7. (5 points) Evaluate:  $\log_5\left(\frac{25x^2}{\sqrt{y}}\right)$ , given  $\log_5(x) = 3$  and  $\log_5(y) = 10$ .

$$\begin{aligned}\log_5\left(\frac{25x^2}{\sqrt{y}}\right) &= \log_5(25) + \log_5(x^2) - \log_5(\sqrt{y}) \\ &= 2 + 2\log_5(x) - \frac{1}{2}\log_5(y) \\ &= 2 + 2*3 - \frac{1}{2}*10 \\ &= 2 + 6 - 5 = \boxed{3}\end{aligned}$$

8. (4 points) You deposit \$4,000 at 8% compounded quarterly for 6 years. How much will be in your account after 6 years?

*Note: you must clearly set up the problem, but you do not need evaluate the expression.*

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$P = 4000$$

$$r = 0.08$$

$$n = 4$$

$$t = 6$$

$$A = 4000\left(1 + \frac{0.08}{4}\right)^{4 \cdot 6}$$

$$A = 4000(1.02)^{24}$$

9. (6 points) Solve:  $\log_2(x+2) + \log_2(x+3) = 1$

$$\log_2((x+2)(x+3)) = 1$$

$$\Rightarrow 2^1 = (x+2)(x+3)$$

$$\Rightarrow 2 = x^2 + 5x + 6$$

$$\Rightarrow x^2 + 5x + 4 = 0 \Rightarrow (x+4)(x+1) = 0$$

$$x = -1, x = -4 ?$$

Check @  $x = -1$

$$\begin{aligned} \log_2(-1+2) + \log_2(-1+3) \\ = \log_2(1) + \log_2(2) \\ = 0 + 1 = 1 \quad \checkmark \end{aligned}$$

@  $x = -4$

$$\log_2(-4+2) + \log_2(-4+3)$$

is DNE

$$x = -1$$

10. (5 points) Sketch:  $y = \log_2(x)$ . Label any intercepts, label the axes and the curve, and identify and include a total of at least four points on your graph for full credit.

$x$	$y$
$\frac{1}{2}$	-1
1	0
2	1
4	2

