Math 1113 **Exam #4** Initials:

Name: Key GTID:

Answer the questions in the spaces provided and put your answer in the $\boxed{\text{BOX}}$. Organize work clearly and simplify answers for full credit.

1a. Find the exact values: $\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$ and $\sec\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$

Formula:

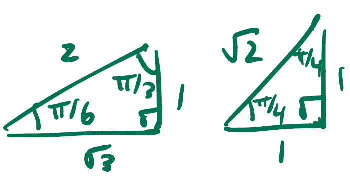
$\cos(\theta+\alpha)=\cos\theta\cos\alpha-\sin\theta\sin\alpha$

$\cos\theta = \boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$

$\sec\theta = \boxed{-\sqrt{2}-\sqrt{6}}$

$$\cos\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\cos\left(\frac{\pi}{3}\right)\cos\left(\frac{\pi}{4}\right)-\sin\left(\frac{\pi}{3}\right)\sin\left(\frac{\pi}{4}\right)$$

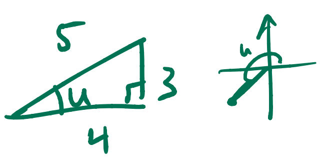
$$=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}-\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$



$$=\boxed{\frac{\sqrt{2}-\sqrt{6}}{4}}$$

$$\sec\left(\frac{\pi}{3}+\frac{\pi}{4}\right)=\frac{4}{\sqrt{2}-\sqrt{6}}\cdot\frac{\sqrt{2}+\sqrt{6}}{\sqrt{2}+\sqrt{6}}=\frac{4\sqrt{2}+4\sqrt{6}}{2-6}=\frac{4\sqrt{2}+4\sqrt{6}}{-4}=\boxed{-\sqrt{2}-\sqrt{6}}$$

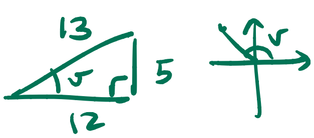
1b. Given that $\tan u=\frac{3}{4}$, with u in quadrant III, and $\sin v=\frac{5}{13}$, with v in quadrant II, find the exact value of $\sin(u+v)$.



Formula:

$\sin(u+v)=\sin u\cos v+\cos u\sin v$

$\boxed{16/65}$



$$=\left(\frac{-3}{5}\right)\left(\frac{-12}{13}\right)+\left(\frac{-4}{5}\right)\left(\frac{5}{13}\right)$$

$$=\frac{36}{65}-\frac{20}{65}=\boxed{\frac{16}{65}}$$

1c. Find the exact value: $2\sin\left(-\frac{\pi}{8}\right)\cos\left(-\frac{\pi}{8}\right)$

$\boxed{-\sqrt{2}/2}$

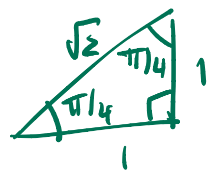
Formula: $\sin 2\theta = 2\sin\theta\cos\theta$

$$2\sin\left(-\frac{\pi}{8}\right)\cos\left(-\frac{\pi}{8}\right) = \sin\left(2 * -\frac{\pi}{8}\right)$$

$$= \sin\left(-\frac{\pi}{4}\right)$$

$$= -\sin(\pi/4) \quad \text{b/c } y=\sin x \text{ is } \underline{\underline{\text{ODD}}}$$

$$= \boxed{-\frac{\sqrt{2}}{2}}$$



1d. Use the half angle identity to find: sin(22.5°)

$\boxed{\dfrac{\sqrt{2-\sqrt{2}}}{2}}$

Formula:

$$\sin\left(\frac{\theta}{2}\right) = \pm\sqrt{\frac{1-\cos\theta}{2}}$$

$$\sin(22.5°) = \sqrt{\frac{1-\cos(45°)}{2}} = \sqrt{\frac{1-\frac{\sqrt{2}}{2}}{2}}$$

$$= \sqrt{\frac{2-\sqrt{2}}{2}\cdot\frac{1}{2}} = \boxed{\frac{\sqrt{2-\sqrt{2}}}{2}}$$

1e. Simplify: $(1 + \tan\theta)(1 - \tan\theta) + \sec^2\theta$

$$(1+\tan\theta)(1-\tan\theta)+\sec^2\theta$$

$$= 1-\tan^2\theta+\sec^2\theta$$

$$= 1-\cancel{\tan^2}\theta+\cancel{\tan^2}\theta+1$$

$$= \boxed{2}$$

$$\boxed{2}$$

Formula:

$\sin^2\theta+\cos^2\theta=1$

$\Rightarrow \tan^2\theta+1=\sec^2\theta$

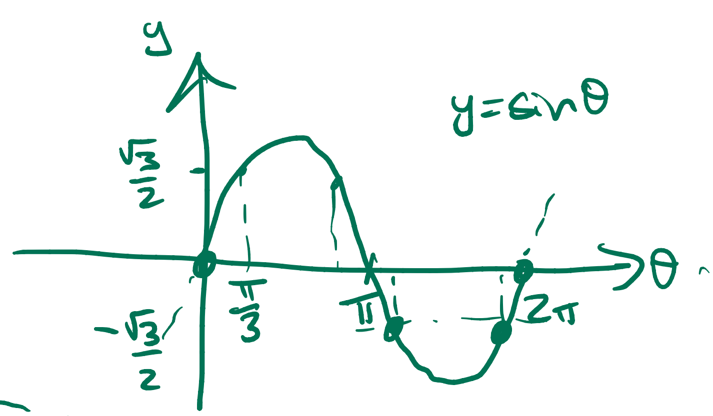
1f. Find all solutions in the interval $[0, 2\pi]$ of: $\sin 2x = -\frac{\sqrt{3}}{2}$

$$\boxed{\frac{2\pi}{3}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{6}}$$

set $\theta = 2x$

$$\theta = \pi + \frac{\pi}{3} = \frac{4\pi}{3}$$

$$\theta = 2\pi - \frac{\pi}{3} = \frac{5\pi}{3}$$



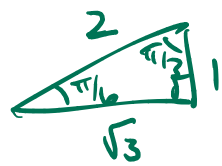
also next loop

$$\theta = \frac{4\pi}{3}+2\pi = \frac{10\pi}{3}$$

So $x = \frac{\theta}{2}$

$$\theta = \frac{5\pi}{3}+2\pi = \frac{11\pi}{3}$$

$$\boxed{x = \frac{4\pi}{6}, \frac{5\pi}{6}, \frac{5\pi}{3}, \frac{11\pi}{3}}$$



2. Use power-reducing formulas to rewrite the expression so that it does not contain trigonometric functions of power greater than 1.

$$\cos^4 x$$

Formula: $\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$$\cos^4\theta = (\cos^2\theta)^2 = \left(\frac{1+\cos 2\theta}{2}\right)^2 = \frac{1+2\cos 2\theta+\cos^2 2\theta}{4}$$

$$= \frac{1}{4}\left(1+2\cos 2\theta + \frac{1+\cos 4\theta}{2}\right) = \frac{1}{4}\left(1+2\cos 2\theta + \frac{1}{2}+\frac{1}{2}\cos 4\theta\right)$$

$$= \boxed{\frac{3}{8}+\frac{1}{2}\cos 2\theta + \frac{1}{8}\cos 4\theta}$$

3. Verify the identity.

$$\tan^2 x - \sin^2 x = \sin^4 x \sec^2 x$$

$\tan^2 x - \sin^2 x$

$= \frac{\sin^2 x}{\cos^2 x} - \sin^2 x$

$= \frac{\sin^2 x - \sin^2 x\cos^2 x}{\cos^2 x}$

$= \frac{\sin^2 x\,(1-\cos^2 x)}{\cos^2 x}$

$= \frac{\sin^2 x\cdot \overbrace{\sin^2 x}^{1-\cos^2 x}}{\cos^2 x}$

$= \sin^4 x\cdot\frac{1}{\cos^2 x}$

$= \sin^4 x\sec^2 x$ ✓

Formula:

$\sin^2 x + \cos^2 x = 1$

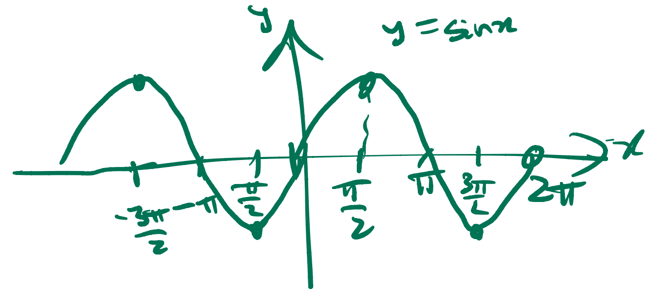
$\Rightarrow 1-\cos^2 x = \sin^2 x$

4. Solve the equation for all values in the range $[-2\pi, 2\pi]$.

$$(1 - \sin x)(1 + \sin x) = 0$$

$1 - \sin x = 0 \Rightarrow \sin x = 1$

$\Rightarrow x = \frac{\pi}{2} + 2\pi n$, n integer



$1 + \sin x = 0 \Rightarrow \sin x = -1$

$\Rightarrow x = \frac{3\pi}{2} + 2\pi n$, n integer.

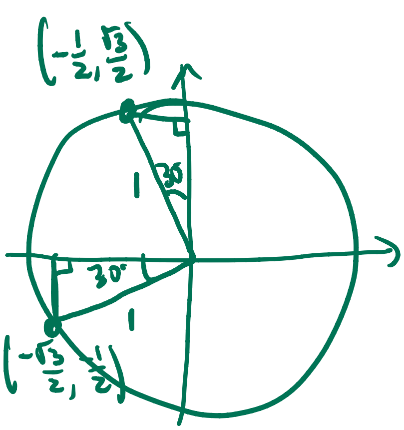
$$\boxed{x = \frac{-3\pi}{2}, \frac{-\pi}{2}, \frac{\pi}{2}, \frac{3\pi}{2}}$$

5. Find the exact value: $\sin(165°) \cdot \cos(45°)$
Hint: use the product-to-sum formula.

Formula:

$$\sin\theta \cos\alpha = \frac{1}{2}[\sin(\theta+\alpha) + \sin(\theta-\alpha)]$$

$$\sin(165°)\cos(45°) = \frac{1}{2}[\sin(165°+45°) + \sin(165°-45°)]$$



$$= \frac{1}{2}[\sin(210°) + \sin(120°)]$$

$$= \frac{1}{2}\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\right) = \boxed{\frac{-1+\sqrt{3}}{4}}$$