Name: Key GTID: ____________

Answer the questions in the spaces provided and put your answer in the $\boxed{\text{BOX}}$. Organize work clearly and simplify answers for full credit.

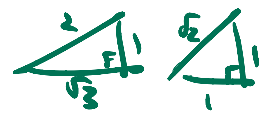
1a. Find the exact value: $\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right)$

$\boxed{\dfrac{\sqrt{2}+\sqrt{6}}{4}}$

Formula: $\cos(\theta - \alpha) = \cos\theta\cos\alpha + \sin\theta\sin\alpha$

$$\cos\left(\frac{\pi}{3} - \frac{\pi}{4}\right) = \cos\frac{\pi}{3}\cos\frac{\pi}{4} + \sin\frac{\pi}{3}\sin\frac{\pi}{4}$$

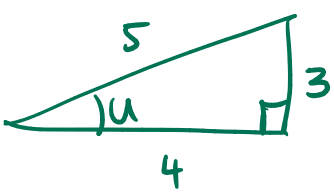
$$= \frac{1}{2}\cdot\frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}\cdot\frac{1}{\sqrt{2}} = \frac{1+\sqrt{3}}{2\sqrt{2}} = \boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

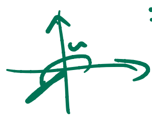


1b. Given that $\tan u = \frac{3}{4}$, with u in quadrant III, and $\sin v = \frac{5}{13}$, with v in quadrant II, find the exact value of $\cos(u + v)$.

Formula: $\cos(\theta + \alpha) = \cos\theta\cos\alpha - \sin\theta\sin\alpha$

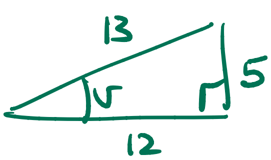
$\boxed{\dfrac{63}{65}}$

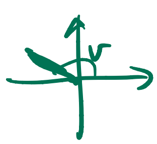




$$= \cos u\cos v - \sin u\sin v$$

$$= \left(-\frac{4}{5}\right)\left(-\frac{12}{13}\right) - \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$$





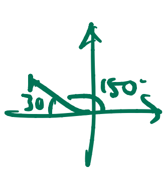
$$= \frac{48}{65} + \frac{15}{65} = \boxed{\frac{63}{65}}$$

1c. Find the exact value: $1 - 2\sin^2 75°$

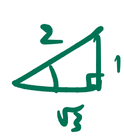
$\boxed{-\sqrt{3}/2}$

Formula: $\cos 2\theta = 1 - 2\sin^2\theta$

$$1 - 2\sin^2 75° = \cos(2 * 75°)$$



$$= \cos(150°)$$



2. Use power-reducing formulas to rewrite the expression so that it does not contain trigonometric functions of power greater than 1.

$$\sin^2 x \cos^2 x$$

Formula: $\sin^2\theta = \frac{1-\cos 2\theta}{2}$

$\cos^2\theta = \frac{1+\cos 2\theta}{2}$

$\sin^2\theta\cos^2\theta = \left(\frac{1-\cos 2\theta}{2}\right)\left(\frac{1+\cos 2\theta}{2}\right)$

$= \frac{1-\cos^2 2\theta}{4}$

$= \frac{1-\left(\frac{1+\cos 4\theta}{2}\right)}{4}$

$= \frac{1}{4}\left(1-\frac{1}{2}-\frac{1}{2}\cos 4\theta\right)$

$= \boxed{\frac{1}{8}-\frac{1}{8}\cos 4\theta}$

3. Verify the identity.

$$(\cos x - \sin x)(\cos x + \sin x) = 1 - 2\sin^2 x$$

Formula: $\sin^2 x + \cos^2 x = 1$

$\Rightarrow \cos^2 x = 1 - \sin^2 x$

$(\cos x - \sin x)(\cos x + \sin x) = \cos^2 x - \sin^2 x$

$= 1 - \sin^2 x - \sin^2 x$

$= 1 - 2\sin^2 x$ ✓